# Population coding and decoding

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# Le traitement du signal appliqué au cerveau

Entrée sensorielle

Bruit sensoriel

Representation neuronale

Bruit neuronal

Commandes motrices

Bruit Moteur

Réponse comportementale

encodage

INFOMAX

decodage

IDEAL OBSERVER

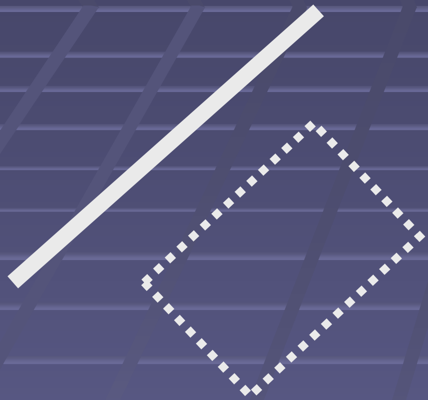
$$x$$

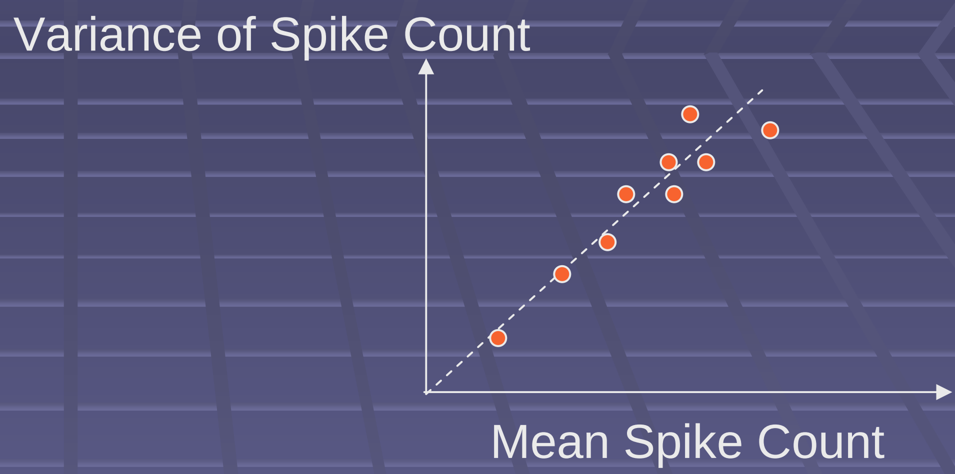
$$s = x + \varepsilon_s$$

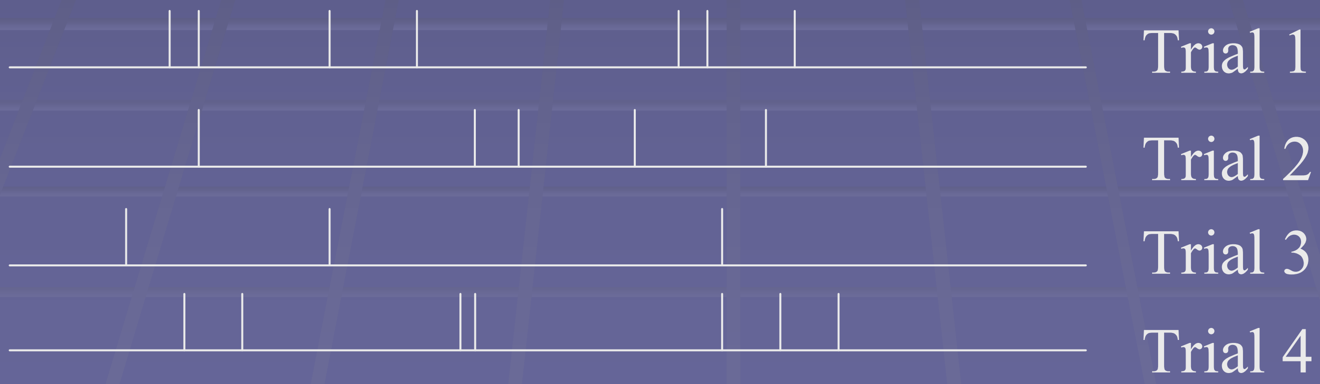
$$r = f\left(s\right) + \varepsilon_n$$

$$\hat{x}$$

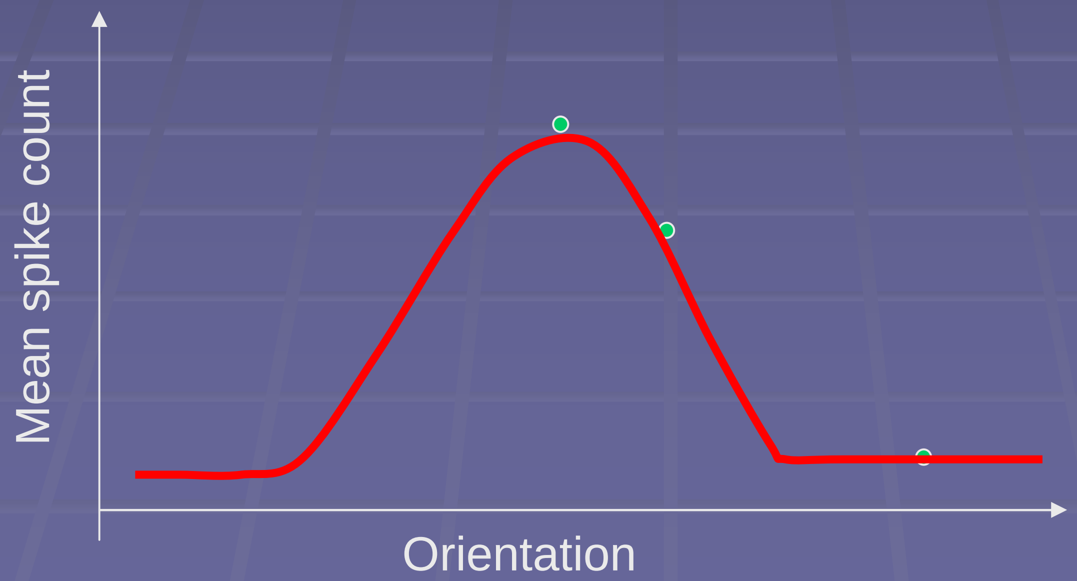
# Poisson Variability in Cortex



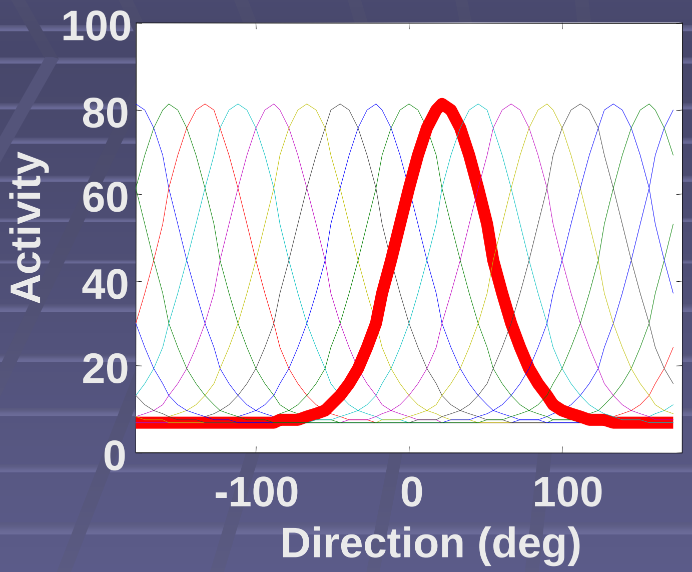


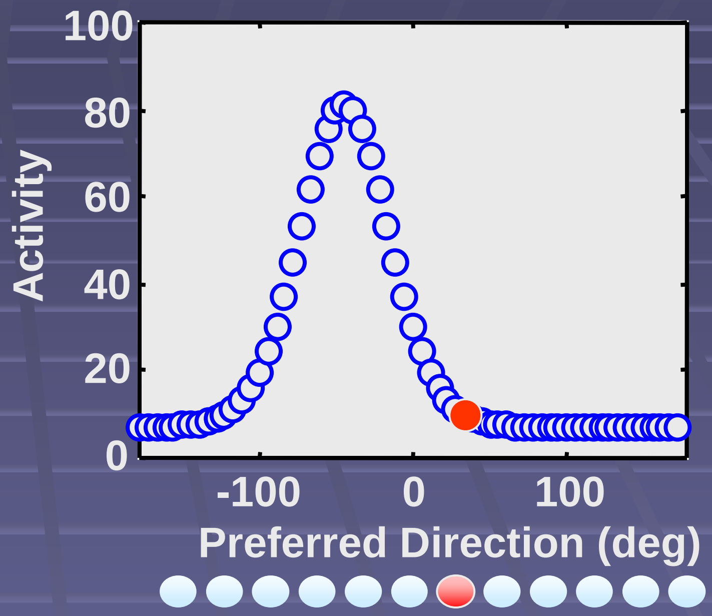
# Tuning curves



# Why "population coding"?




Tuning Curves




Average pattern of activity

$$\langle r_i \rangle = f\left(x - x_i\right) = f_i\left(x\right)$$

# Examples of Population codes

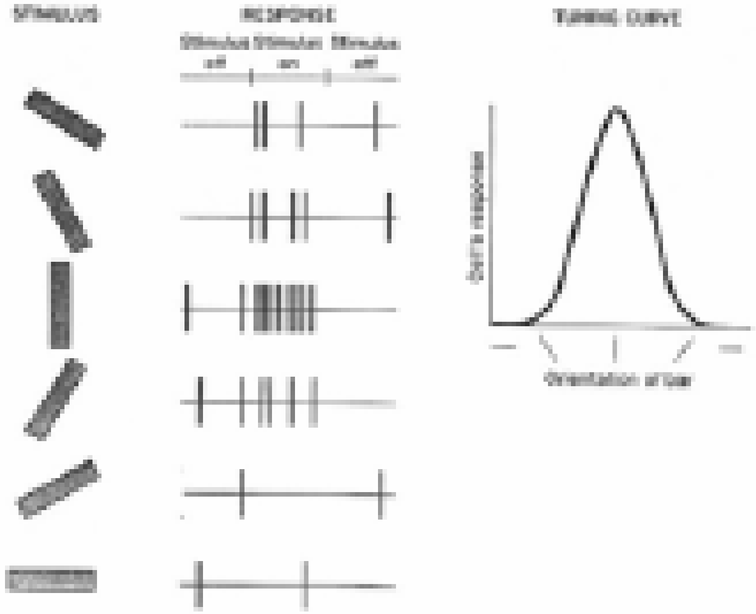



FIGURE 4.8 Response of a single cortical cell to bars presented at various orientations.

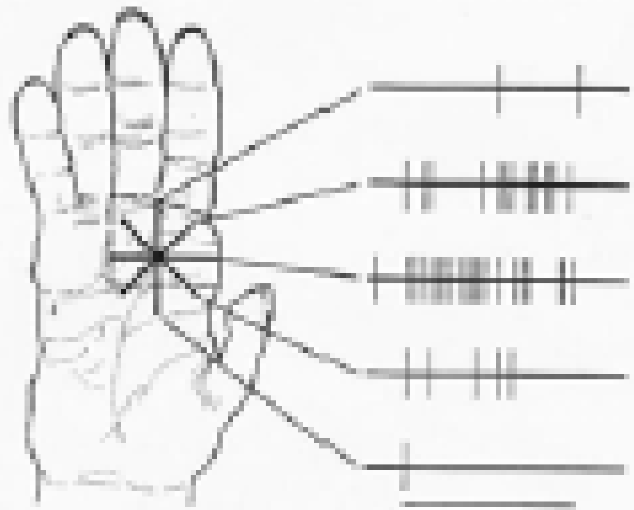
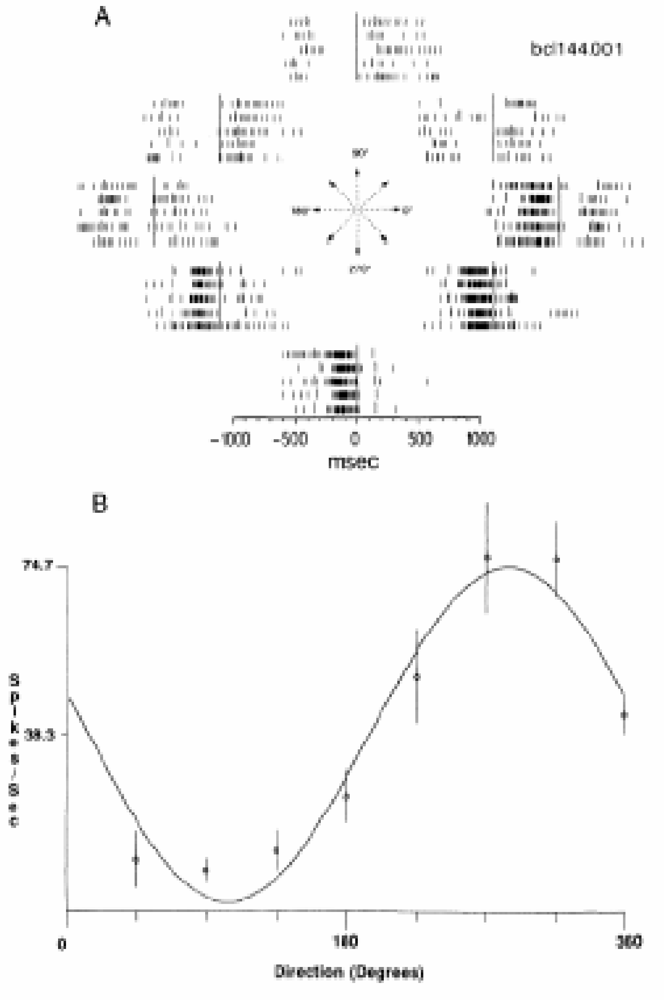


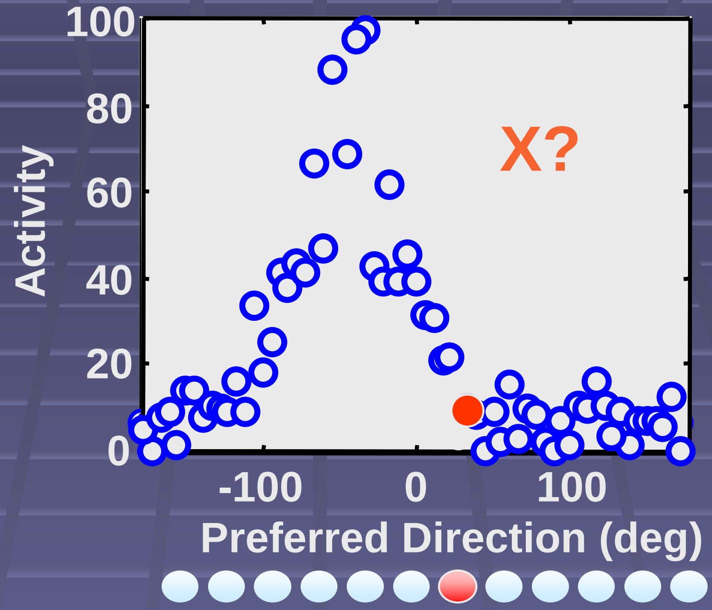
**Figure 11.22**
*The receptive field of a neuron in the monkey's somatosensory cortex that responds when an edge is placed on the hand. This cell responds well when the edge is oriented horizontally but responds less well to other orientations. (From Hyvarinen & Poranen, 1978.)*

# Examples of Population codes



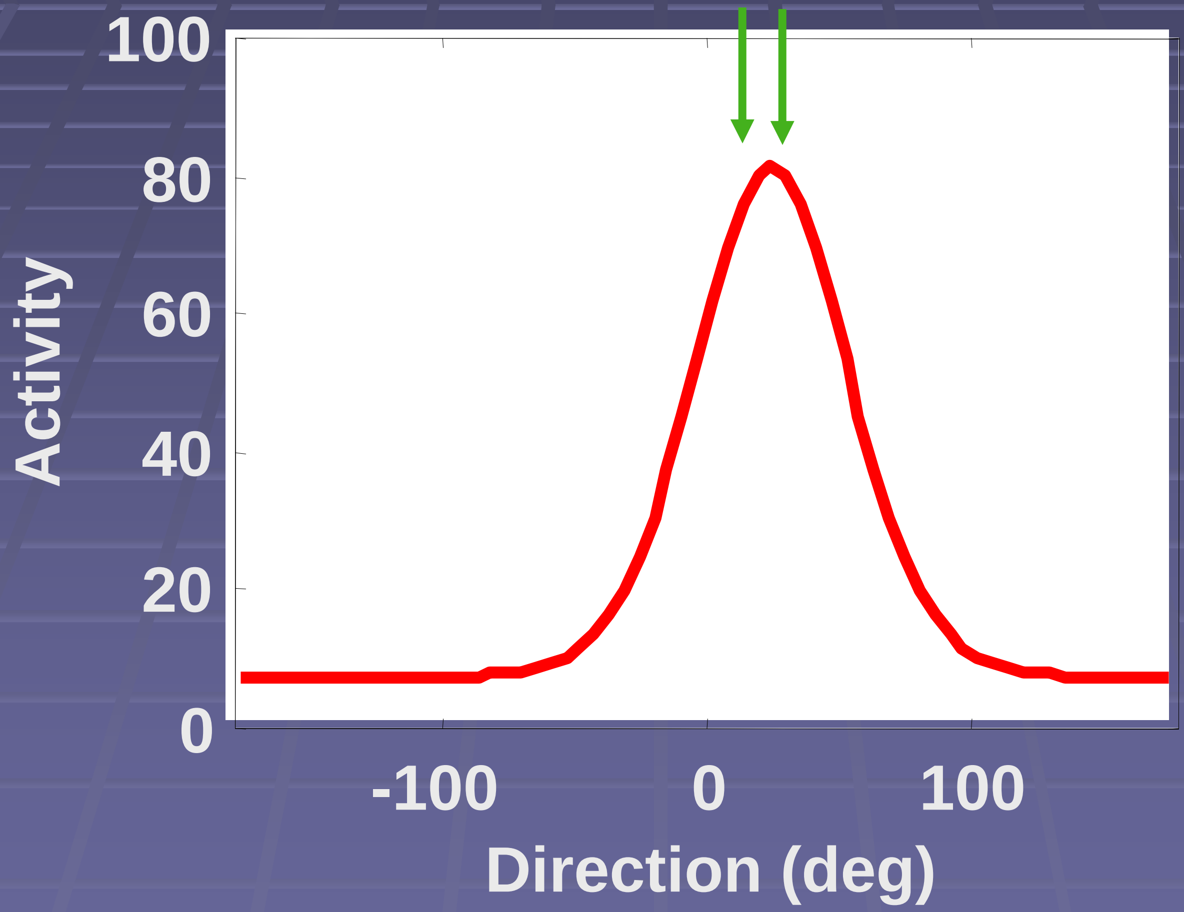

Georgopoulos 1986

# On a single trial: noisy population Code




Pattern of activity (**r**)

# How precisely can we read a noisy population code?



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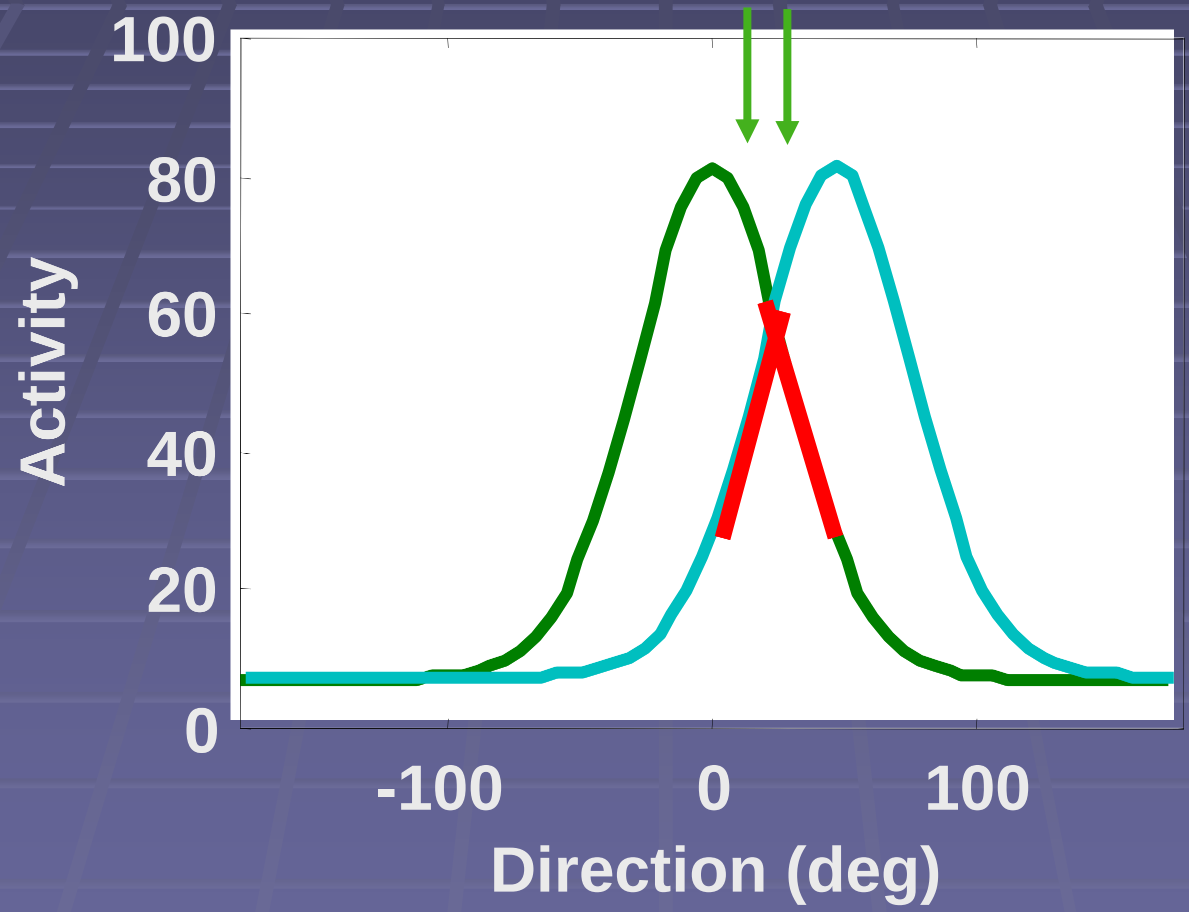
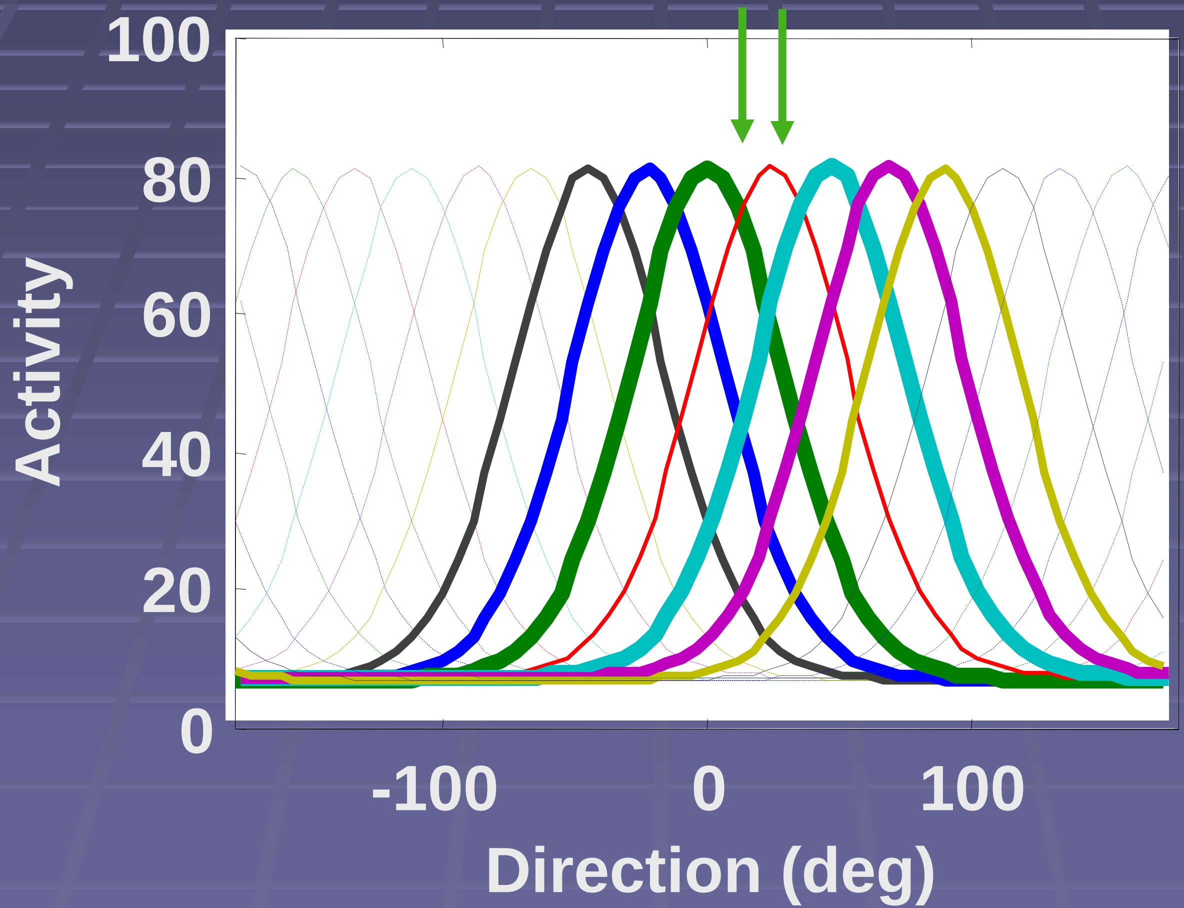


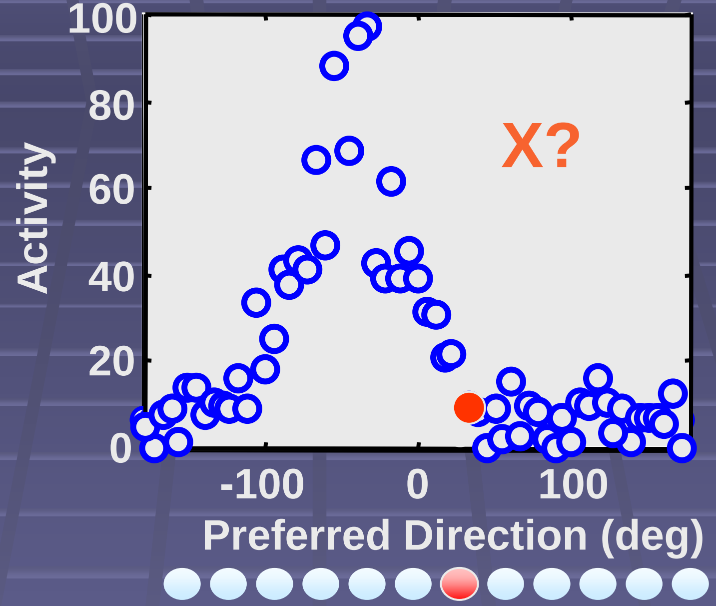

The higher the slope, the better

# How precisely can we read a noisy population code?




The more neuron, the better

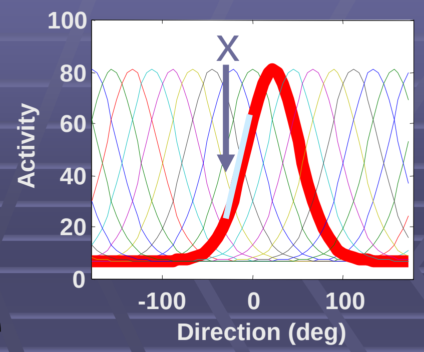
# Fisher information quantifies the performance of the best possible estimator (ideal observer)




Pattern of activity (**r**)

$$\sigma^2 = \left\langle \left( x - \hat{x} \right)^2 \right\rangle = \frac{1}{I(x)}$$

# Fisher Information




- For one neuron with Poisson noise

$$I_i(x) = \frac{f_i'(x)^2}{f_i(x)}$$

Derivative of the tuning curve

Tuning curve (mean activity)

- For *n* independent neurons :

$$I(x) = \sum_i \frac{f_i'(x)^2}{f_i(x)}$$

Large slope is good!

The more neurons, the better!

Small variance is good!

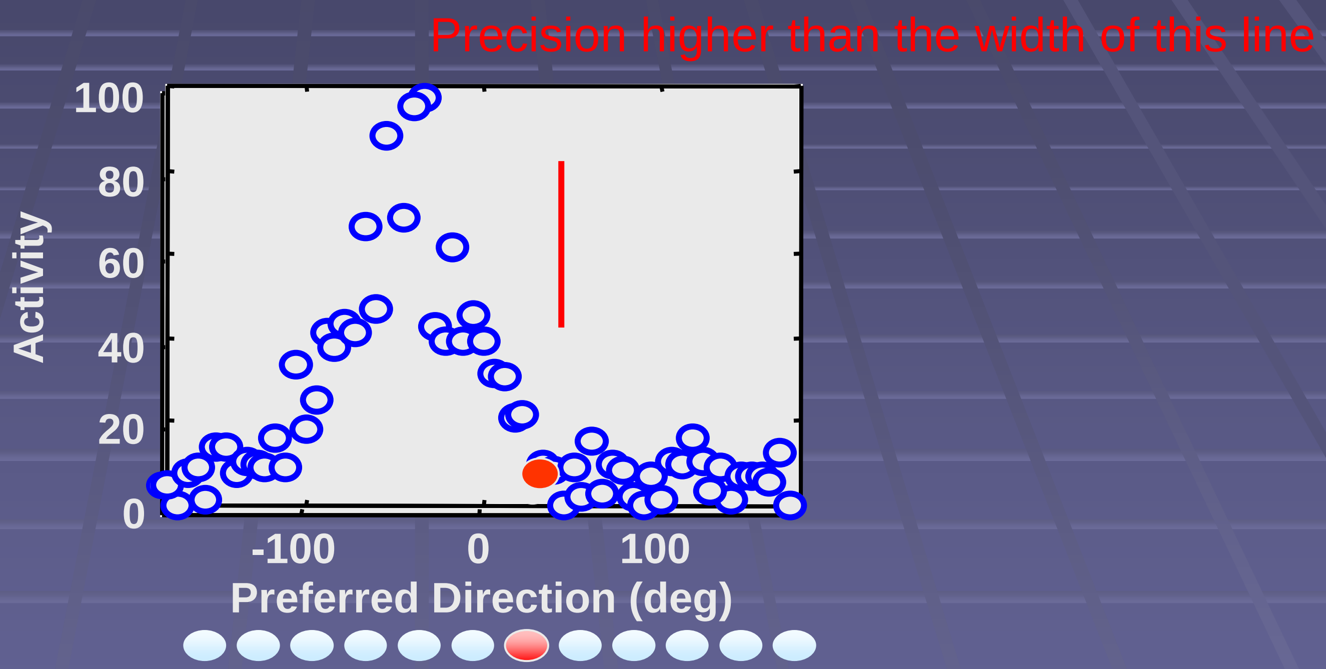
# Variance of the best possible estimator (an ideal observer)

- Population of neuron with independent Poisson noise:

$$\sigma_{ML}^2 = \left\langle \left( x - \hat{x}_{ML} \right)^2 \right\rangle = \frac{1}{\sum_i \frac{f_i'^2}{f_i}}$$

Can the brain reach such a level of performance?

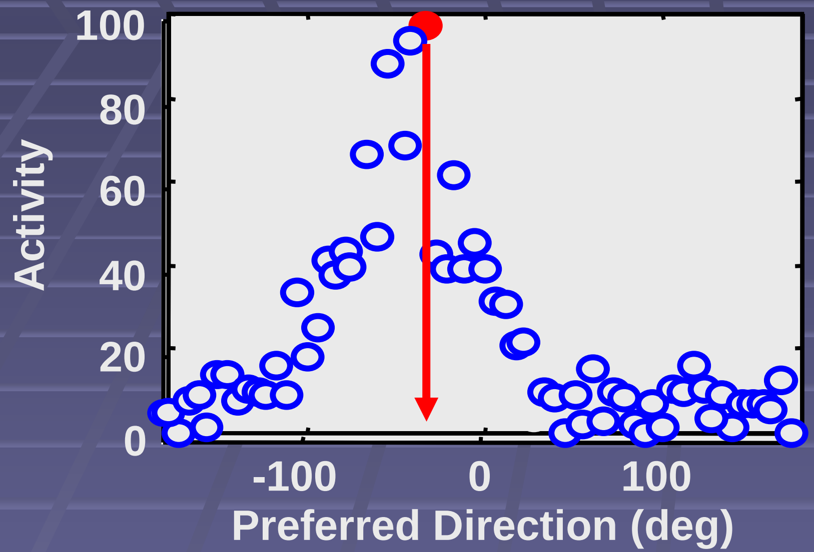
# Implication: very high precision with very noisy neurons



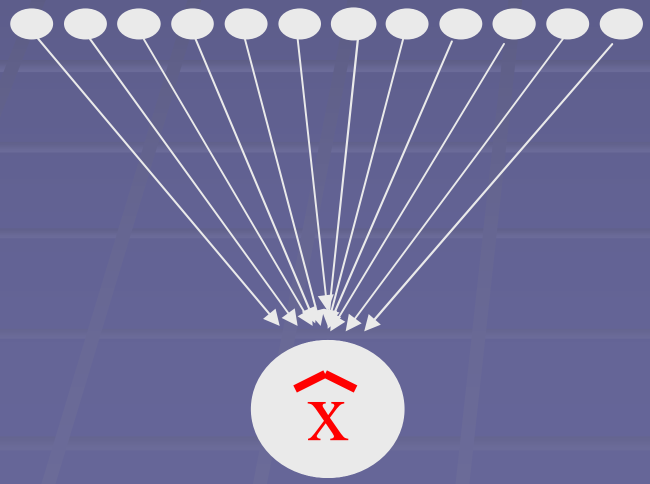

Pattern of activity (**r**)

Hyper-acuity: usually the precision is much better than the distance between two adjacent neuron's preferred x

# What estimator does the brain use?

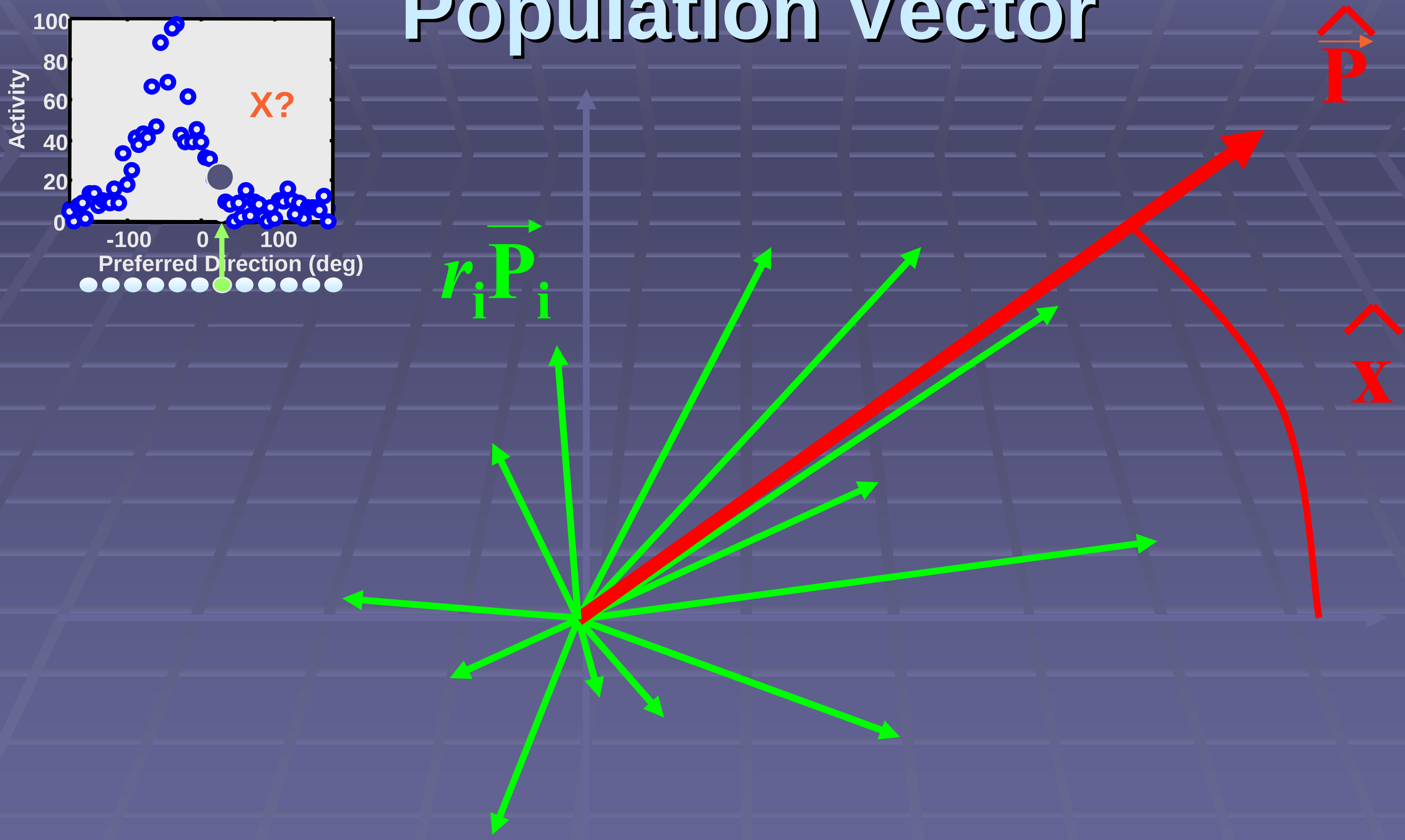



Maximally active unit: terrible!



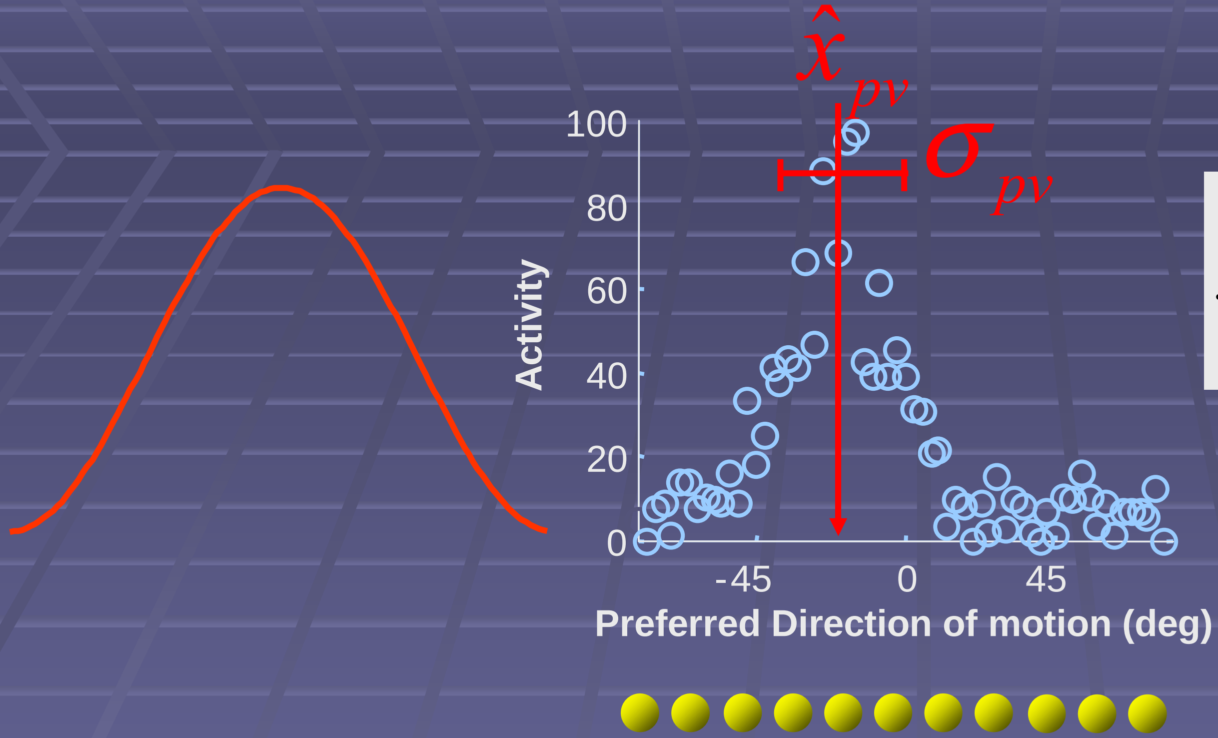

Linear estimator; not so good

# Population Vector




Georgepoulos et al.

# Maximum likelihood



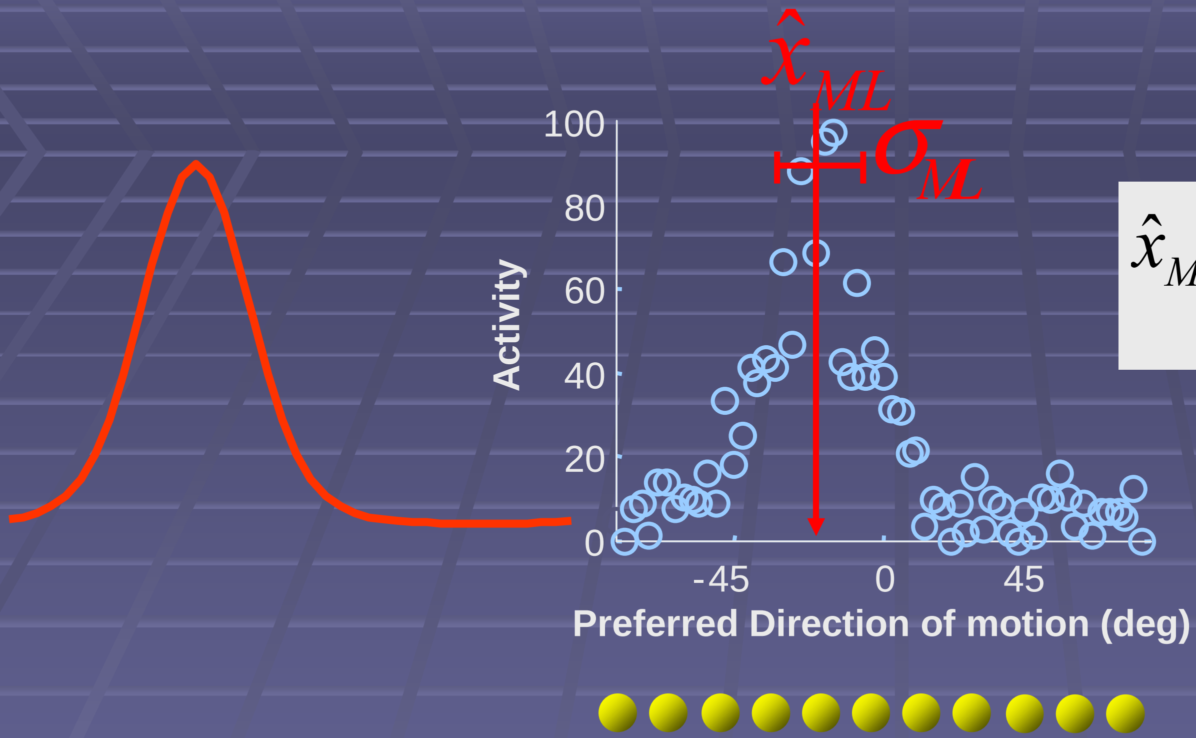

$$\hat{x}_{pv} = \text{angle}\left( \sum_i r_i \bar{P}_i \right)$$

$$\sigma_{pv}^2 = \left\langle \left( x - \hat{x}_{pv} \right)^2 \right\rangle = \frac{1}{I(x)}$$

Maximum performance for an unbiased estimator

Fisher information

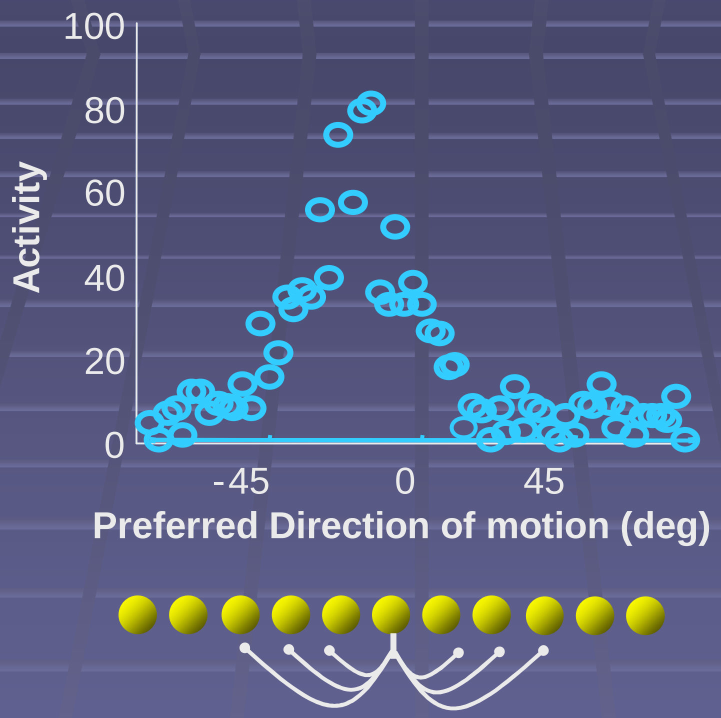
# Maximum likelihood




$$\hat{x}_{ML} = \underset{x}{\arg\max}\left(p\left(\mathbf{r} \mid x\right)\right)$$

$$\sigma^2_{ML} = \frac{1}{I\left(x\right)}$$

# How to decode?



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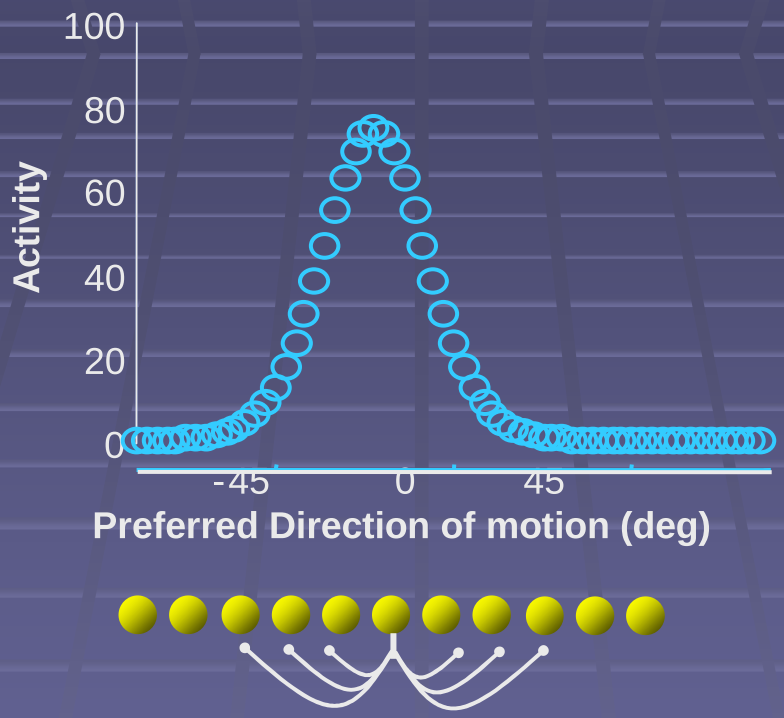
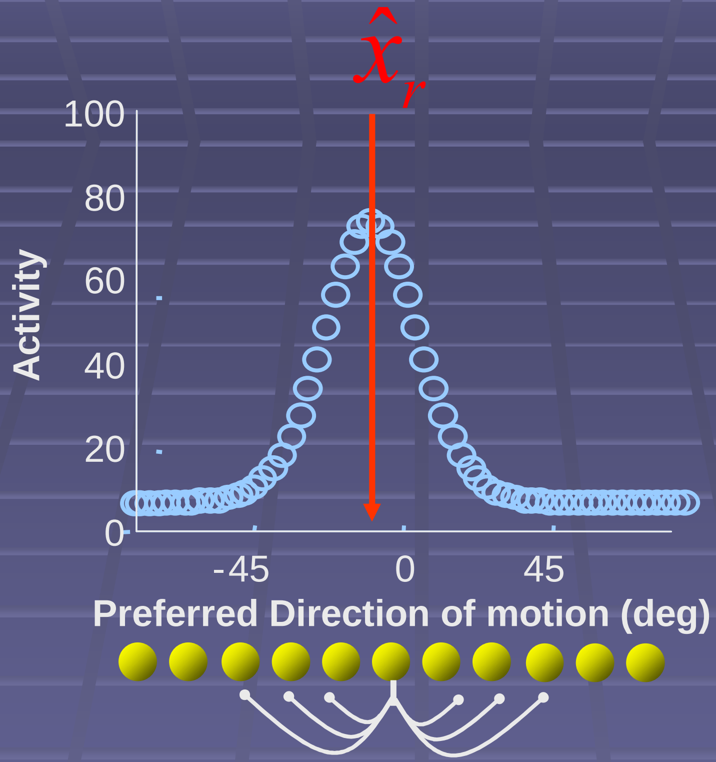
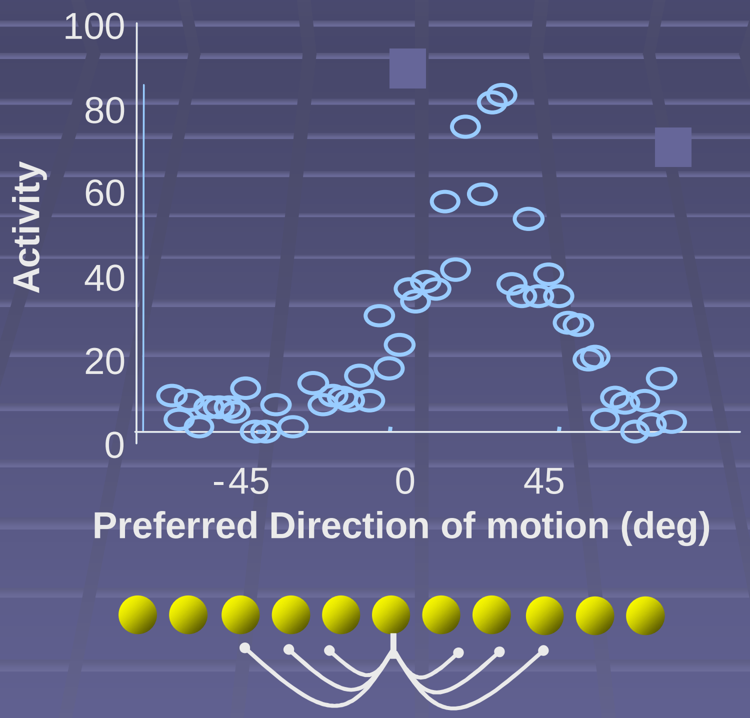


# Recurrent network



# How to decode?



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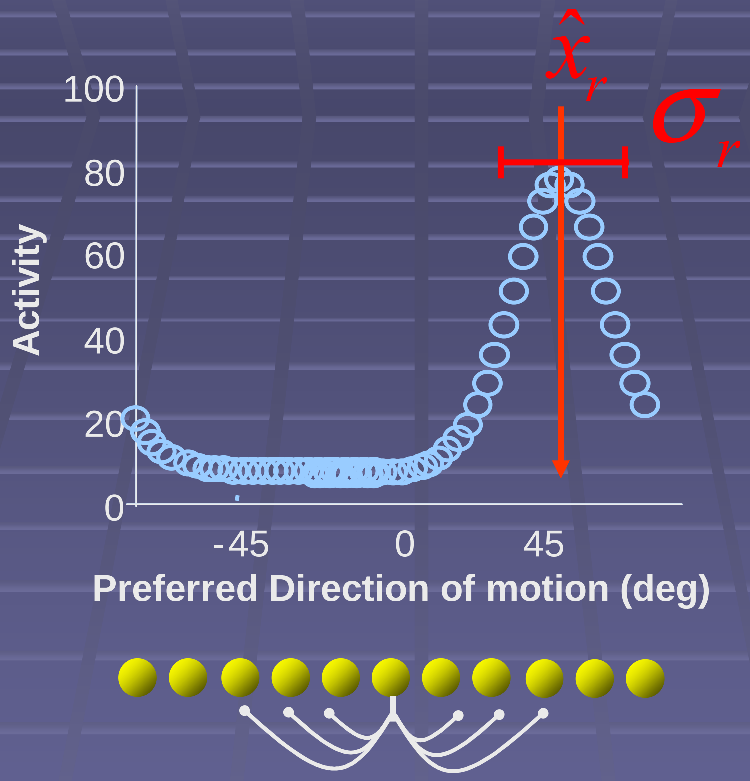
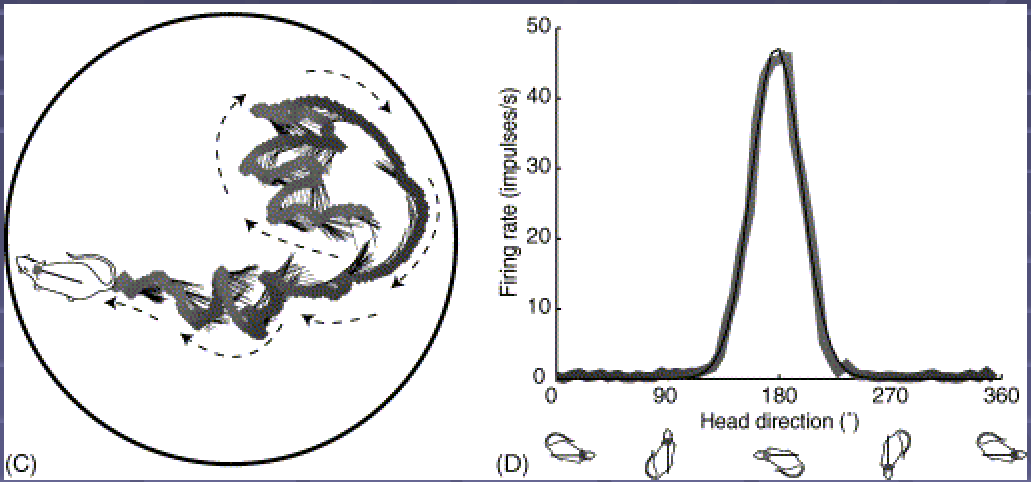
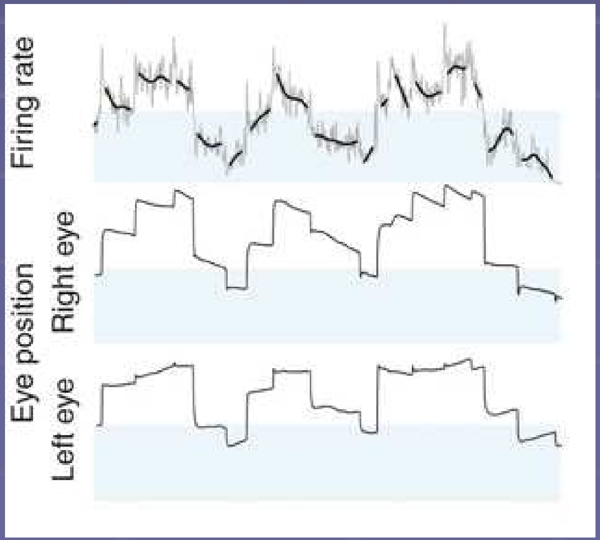


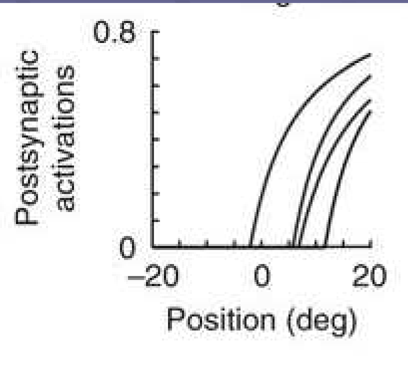
# Exemple of line attractors

Head direction cells in the hippocampus

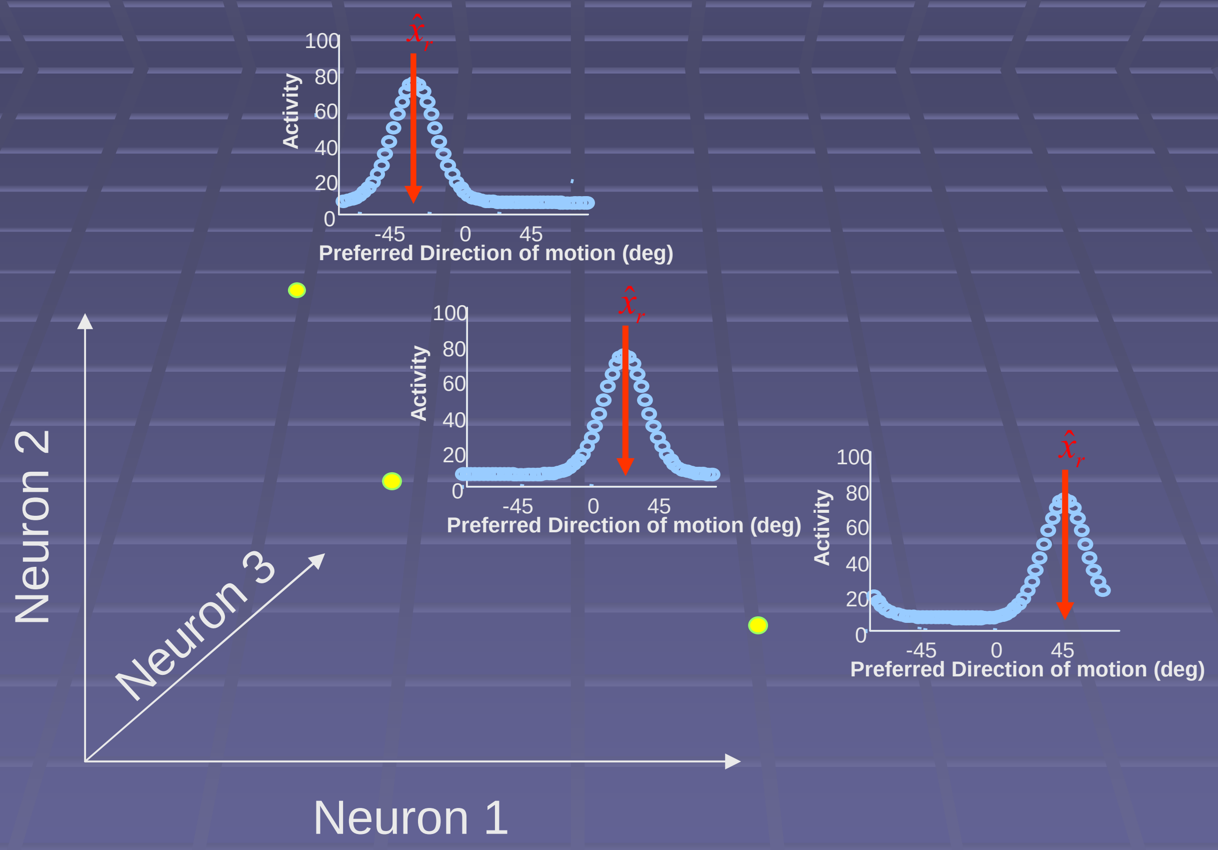



Oculomotor integrator: Graded firing rate as a function of gaze angle



# Line attractor networks



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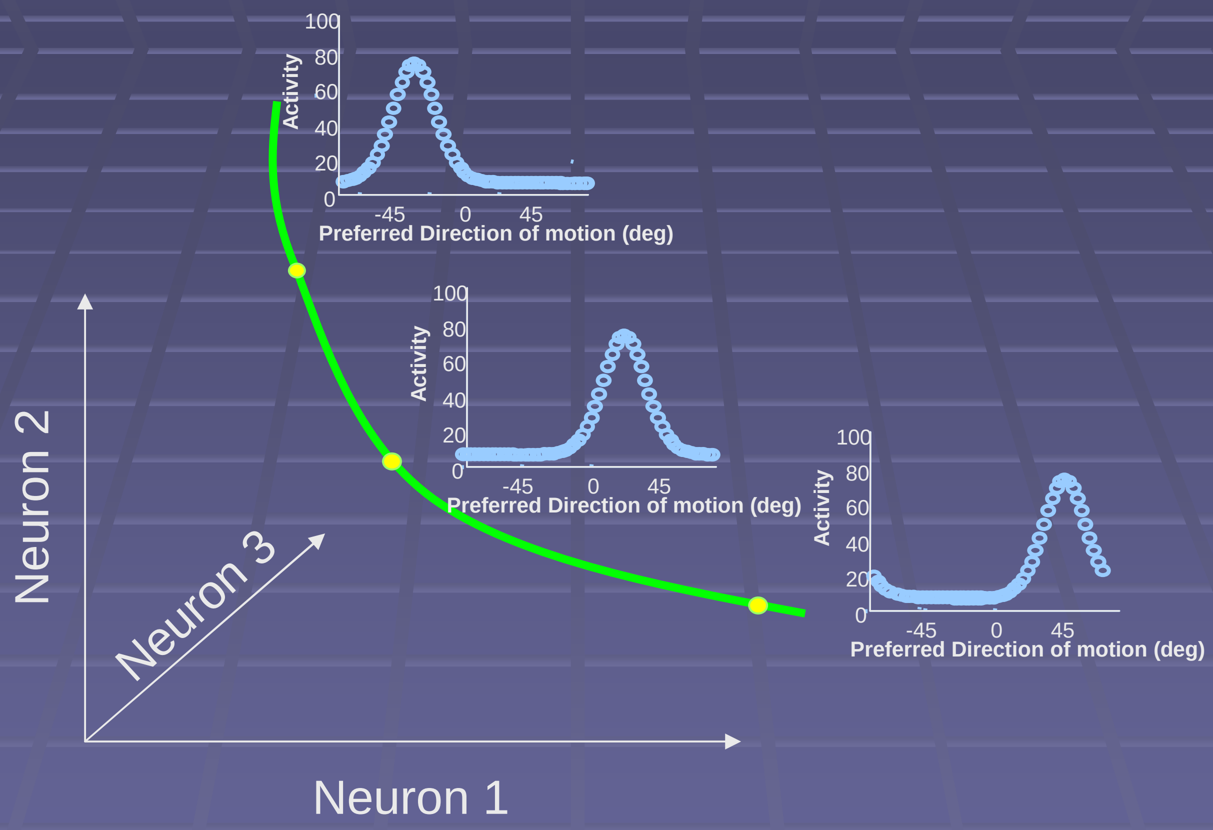

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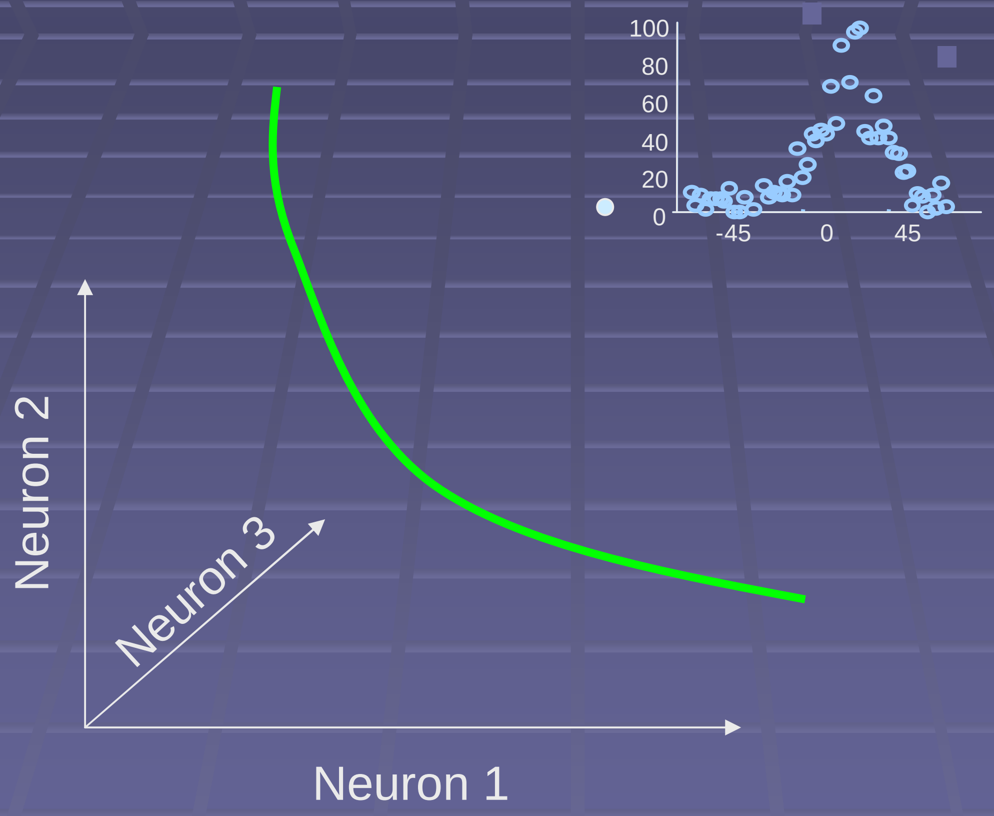

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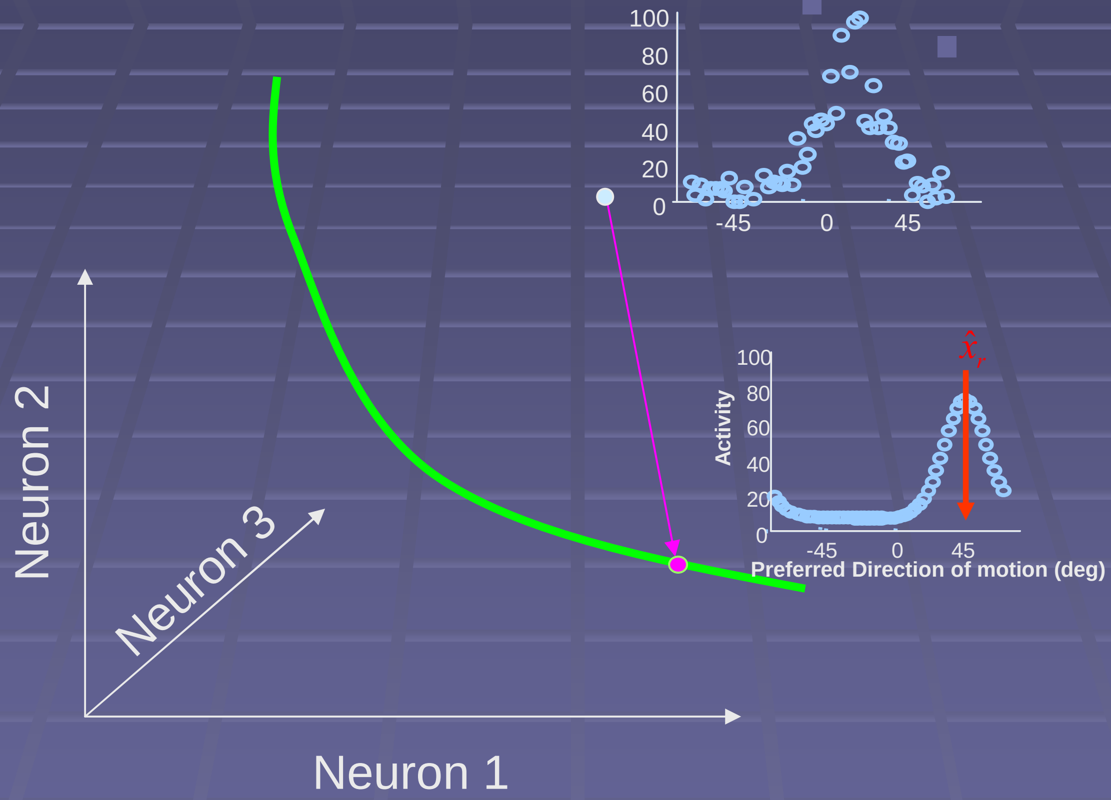

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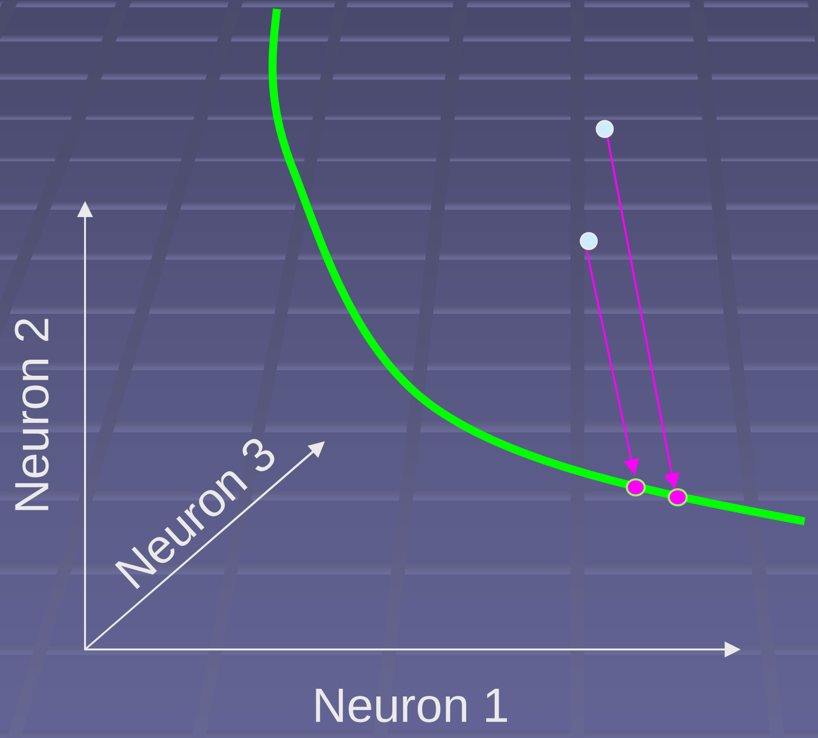

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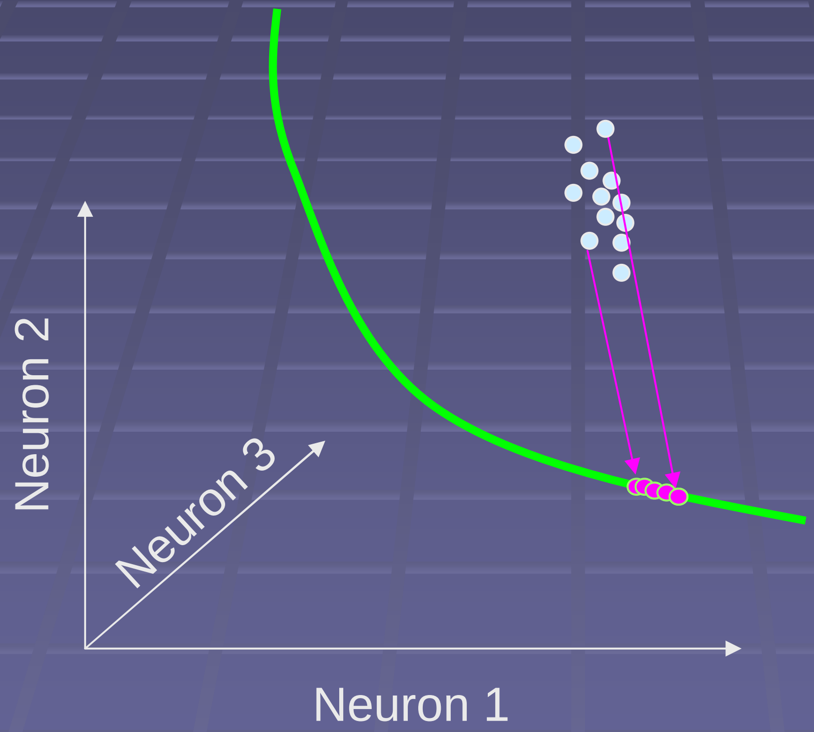

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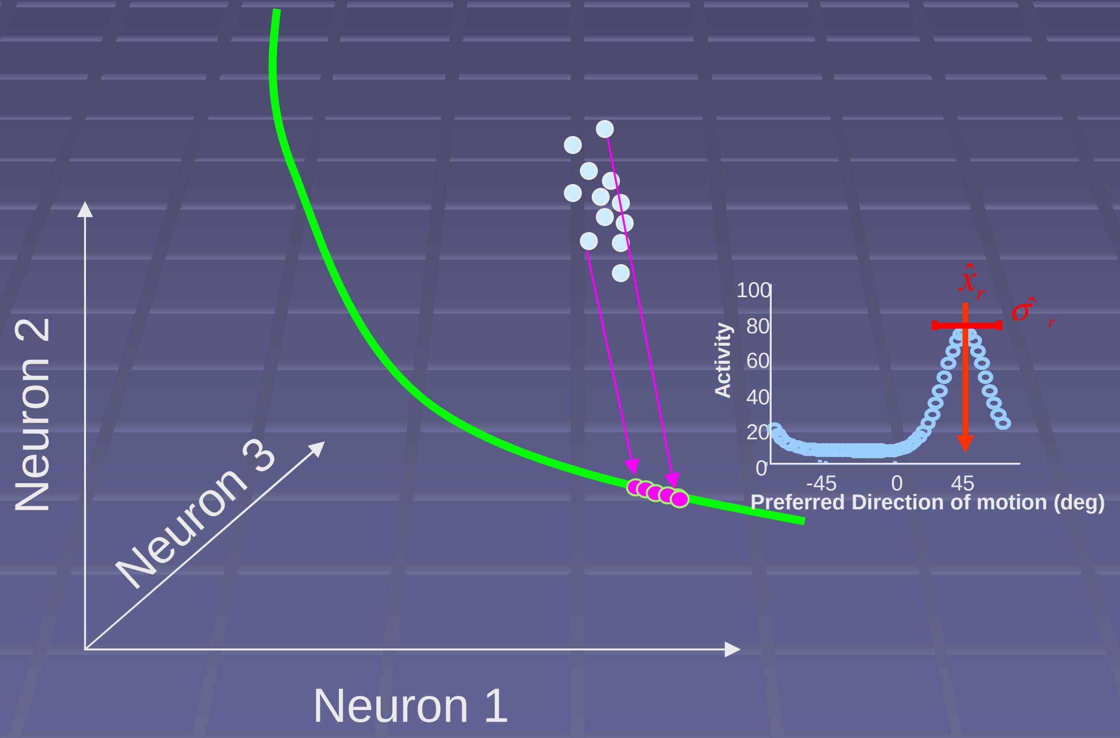

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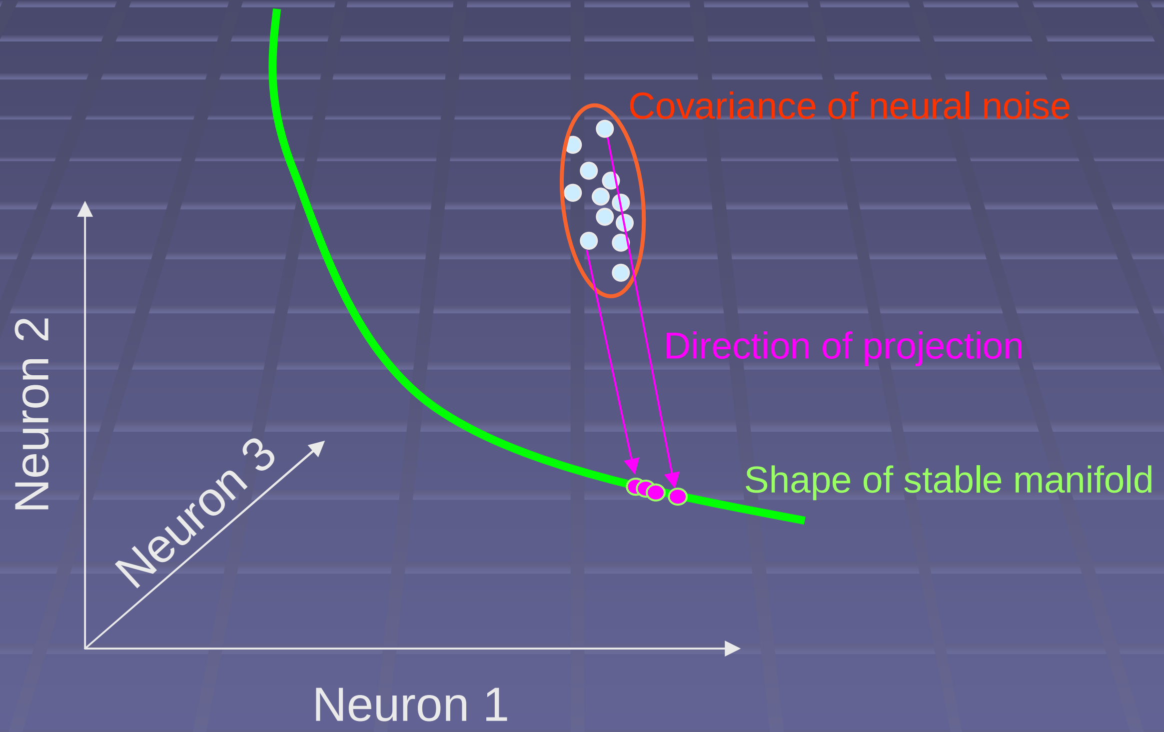

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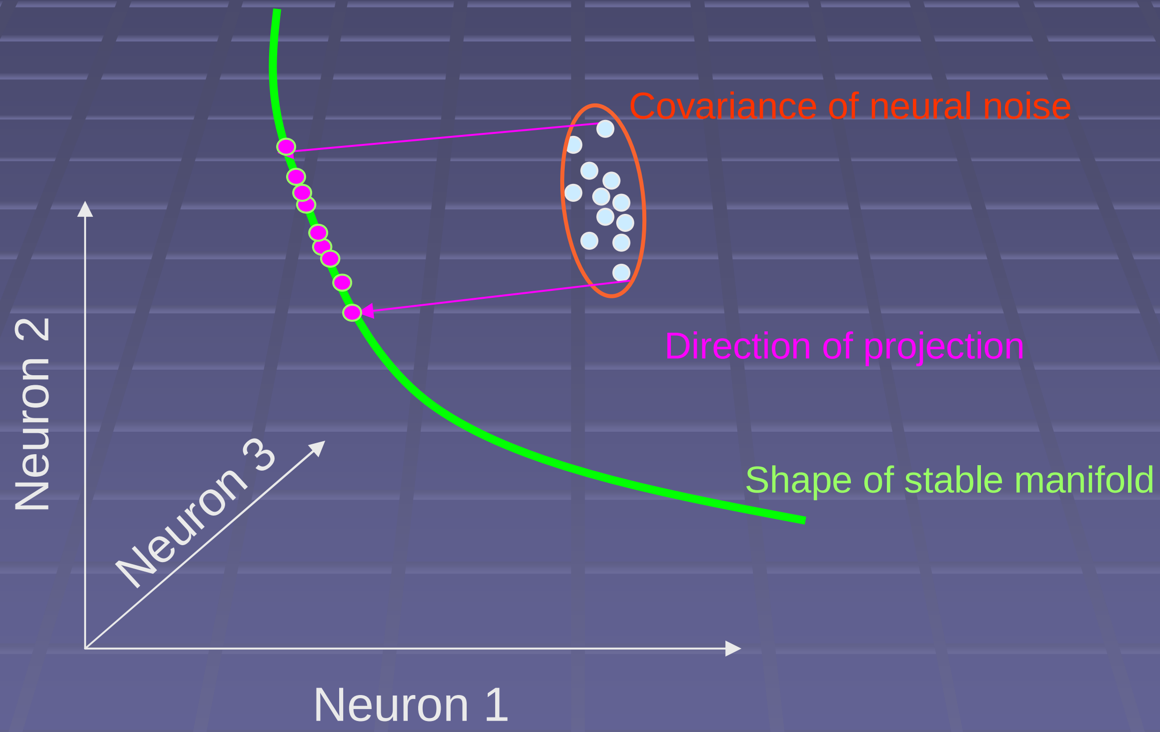

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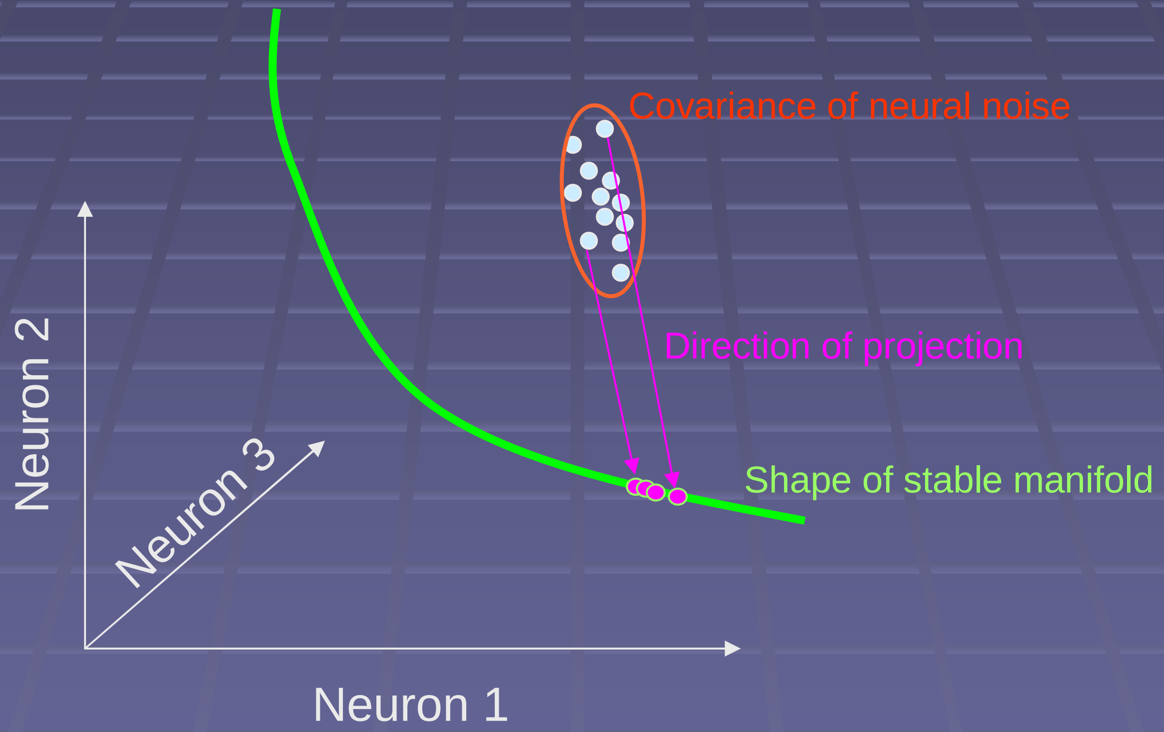


$$\Sigma^{-1} \frac{\partial \mathbf{f}}{dt}^{T} = v(W)$$

# Conclusion

- Sensory and motor variables are represented with noisy population codes.
- It is possible to compute the precision of these codes. It is the inverse of the Fisher information, a function the slope of the tuning curve, the neural noise, and the number of neurons.
- But this information is difficult to get. The most often used methods (linear estimator, population vectors) are not optimal. They are not ideal observers.
- The best one can do is find the variable that maximize the probability of the neural response, the maximum lieklihood estimator.
- The maximum likelihood estimate can be recovered by a line attractor network.

# Problems…

- Persistent Line attractors are rare. There is no trace of them in purely sensory areas.
- Neurons are never noiseless. The smooth stable state does not exist.
- There is no time to converge in the real world.
- How is the attractor reset or updated by new inputs?