Classical conditioning a la Pavlov
Classical conditioning

1. **food** → **salivation**

2. **sound** → **no salivation**

3. **sound** → **food** → **salivation**

4. **sound** → **salivation**
## Extinction

<table>
<thead>
<tr>
<th></th>
<th>Training</th>
<th>Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>sound</td>
<td>food</td>
</tr>
<tr>
<td>2.</td>
<td>sound</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>sound</td>
<td>no food</td>
</tr>
<tr>
<td>4.</td>
<td>sound</td>
<td></td>
</tr>
</tbody>
</table>
What does the dog want?

Assume: The dog wants to be able to predict the reward!

\[ u_i \quad \text{stimulus in trial } i: \quad u_i = 0 \quad \text{or} \quad u_i = 1 \]

\[ r_i \quad \text{reward in trial } i: \quad r_i = 0 \quad \text{or} \quad r_i = 1 \]

\[ v_i \quad \text{reward that the dog expects in trial } i \]
What does the dog learn?

Assume: The dog wants to be able to predict the reward!

- $u_i$: stimulus in trial $i$: $u_i = 0$ or $u_i = 1$
- $r_i$: reward in trial $i$: $r_i = 0$ or $r_i = 1$
- $v_i$: reward that the dog expects in trial $i$

Assume: The dog learns to minimize a “loss” function:

$$L = \sum_{i=1}^{N} (r_i - v_i)^2$$
What does the dog learn?

Assume: The dog wants to be able to predict the reward!

- $u_i$: stimulus in trial $i$: $u_i = 0$ or $u_i = 1$
- $r_i$: reward in trial $i$: $r_i = 0$ or $r_i = 1$
- $v_i$: reward that the dog expects in trial $i$

Assume: The dog learns to minimize a “loss” function:

$$L = \sum_{i=1}^{N} (r_i - v_i)^2$$

Assume:

- dog’s model of the world
  $$v_i = wu_i$$
What does the dog learn?

Assume: The dog wants to be able to predict the reward!

- $u_i$ stimulus in trial $i$: $u_i = 0$ or $u_i = 1$
- $r_i$ reward in trial $i$: $r_i = 0$ or $r_i = 1$
- $v_i$ reward that the dog expects in trial $i$

Assume: The dog learns to minimize a “loss” function:

$$L = \sum_{i=1}^{N} (r_i - v_i)^2$$

Assume: The dog's model of the world

$\theta_i$ parameter that the dog needs to learn from observations
What does the dog learn?

Assume: The dog wants to be able to predict the reward!

- $u_i$: stimulus in trial $i$: $u_i = 0$ or $u_i = 1$
- $r_i$: reward in trial $i$: $r = 0$ or $r_i = 1$
- $v_i$: reward that the dog expects in trial $i$

Assume: The dog learns to minimize a “loss” function:

$$L = \sum_{i=1}^{N} (r_i - v_i)^2$$

$v_i = wu_i$

“Loss” in the $i$-th trial:

$$L_i = (r_i - wu_i)^2$$
How should the dog adopt its world model?

“Loss” in the i-th trial:

\[ L_i = (r_i - wu_i)^2 \]

Update parameter \( w \) to decrease loss!

\[ w \rightarrow w - \epsilon \frac{d}{dw} L_i \]
How should the dog adopt its world model?

\[ \frac{d}{dw} L_i = \frac{d}{dw} (r_i - w u_i)^2 \]

Update parameter \( w \) to decrease loss!

\[ w \rightarrow w - \epsilon \frac{d}{dw} L_i \]
How should the dog adopt its world model?

\[
\frac{d}{dw} L_i = \frac{d}{dw} (r_i - wu_i)^2
\]

\[
= -2u_i (r_i - wu_i)
\]

Update parameter \( w \) to decrease loss!

\[
w \rightarrow w - \epsilon \frac{d}{dw} L_i
\]
How should the dog adopt its world model?

\[ \frac{d}{dw} L_i = \frac{d}{dw} (r_i - wu_i)^2 \]

\[ = -2u_i (r_i - wu_i) \]

\[ = -2u_i \delta_i \]

\[ \delta_i = r_i - uw_i = r_i - v_i \]

“prediction error”

Update parameter \( w \) to decrease loss!

\[ w \rightarrow w - \epsilon \frac{d}{dw} L_i \]
Minimizing the loss function

Minimize loss function (= maximize ability to predict reward)

\[ \frac{d}{dw} L_i = \frac{d}{dw} (r_i - wu_i)^2 \]

\[ = -2u_i (r_i - wu_i) \]

\[ = -2u_i \delta_i \]

“Rescorla-Wagner”-rule

\[ w \rightarrow w + \epsilon \delta_i u_i \]

\[ \delta_i = r_i - uw_i = r_i - v_i \]

“prediction error”

\[ \frac{d}{dw} L_i \]
Rescorla-Wagner rule: Conditioning and extinction

stimulus + reward
(CS + US)

$u_i = 1 \quad r_i = 1$

stimulus
(CS)

$u_i = 1 \quad r = 0$

$v_i = w u_i$
Rabbit eye blinking: Conditioning and extinction

Schneiderman et al, Science, 1962
Classical conditioning: blocking

1. Training:
   - Sound → Food → Salivation

2. Training:
   - Clapping → Sound → Food → Salivation

3. Testing:
   - Sound → Salivation

4. Testing:
   - Clapping → No Salivation
Reward prediction with multiple stimuli: vectorized Rescorla-Wagner rule

Simple linear model (i-th trial)

\[ v_i = w \cdot u_i \]

“Rescorla-Wagner”-rule

\[ w \rightarrow w + \epsilon \delta u_i \]

\[ \delta = r_i - v_i \]

“prediction error”
Classical conditioning: blocking

\[ v_i = w \cdot u_i \]

\[ w \rightarrow w + \epsilon \delta u_i \]
## Inhibitory conditioning

<table>
<thead>
<tr>
<th></th>
<th>Inhibitory conditioning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>1.</strong> sound → food → salivation</td>
</tr>
<tr>
<td>training</td>
<td><strong>2.</strong> clapping → food → salivation</td>
</tr>
<tr>
<td></td>
<td><strong>3.</strong> light sound → salivation disappears</td>
</tr>
<tr>
<td>testing</td>
<td><strong>4.</strong> clapping → salivation</td>
</tr>
<tr>
<td></td>
<td><strong>5.</strong> light clapping → reduced salivation</td>
</tr>
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</table>
Classical conditioning:
inhibitory conditioning

![Graph showing classical conditioning with inhibitor conditioning.](image-url)
Secondary conditioning

1. sound -> food -> salivation
2. clapping -> sound -> food -> salivation
3. sound -> salivation
4. clapping -> salivation

→ cannot be explained by Rescorla-Wagner ...