The Rescorla-Wagner learning rule

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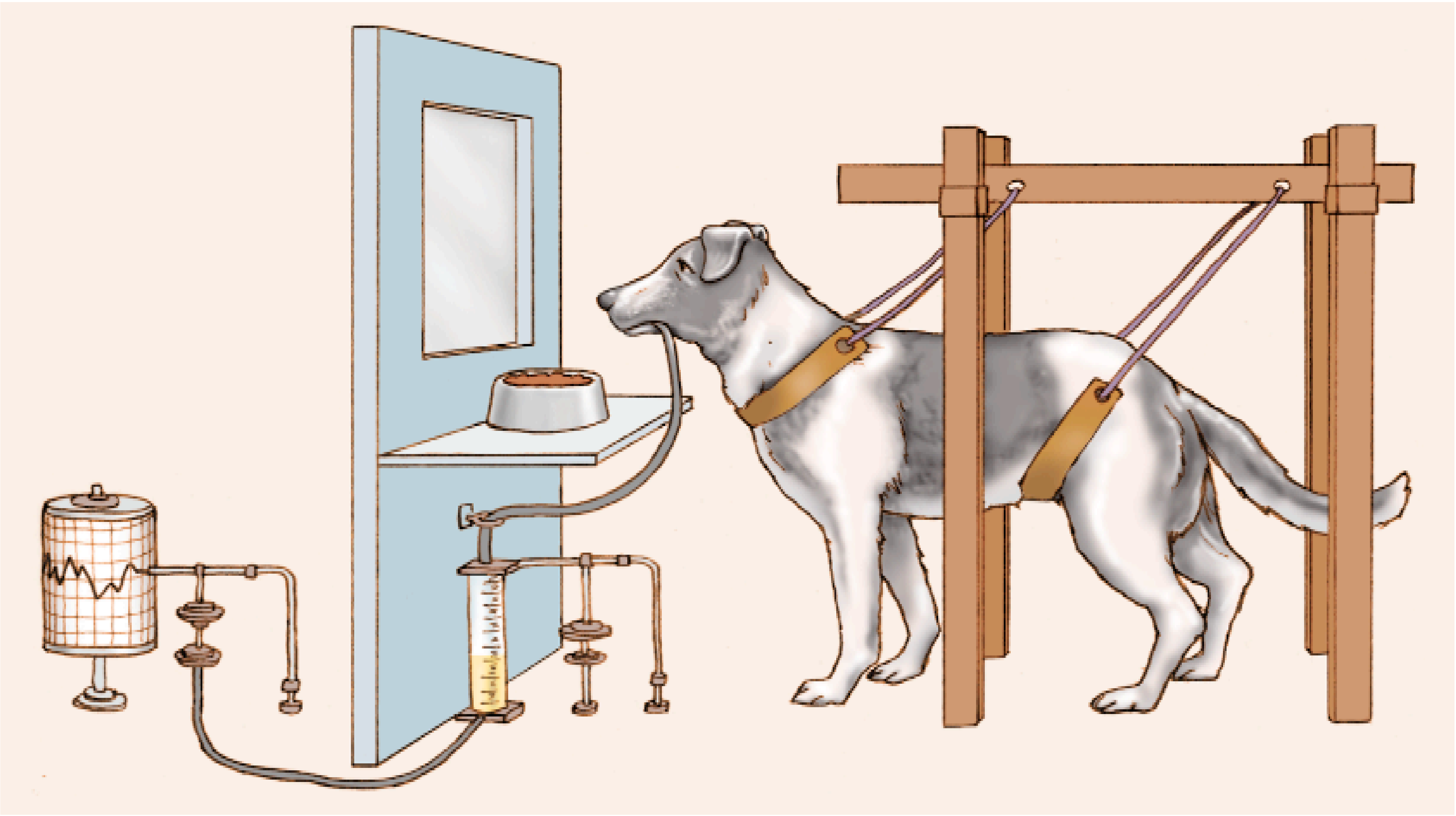
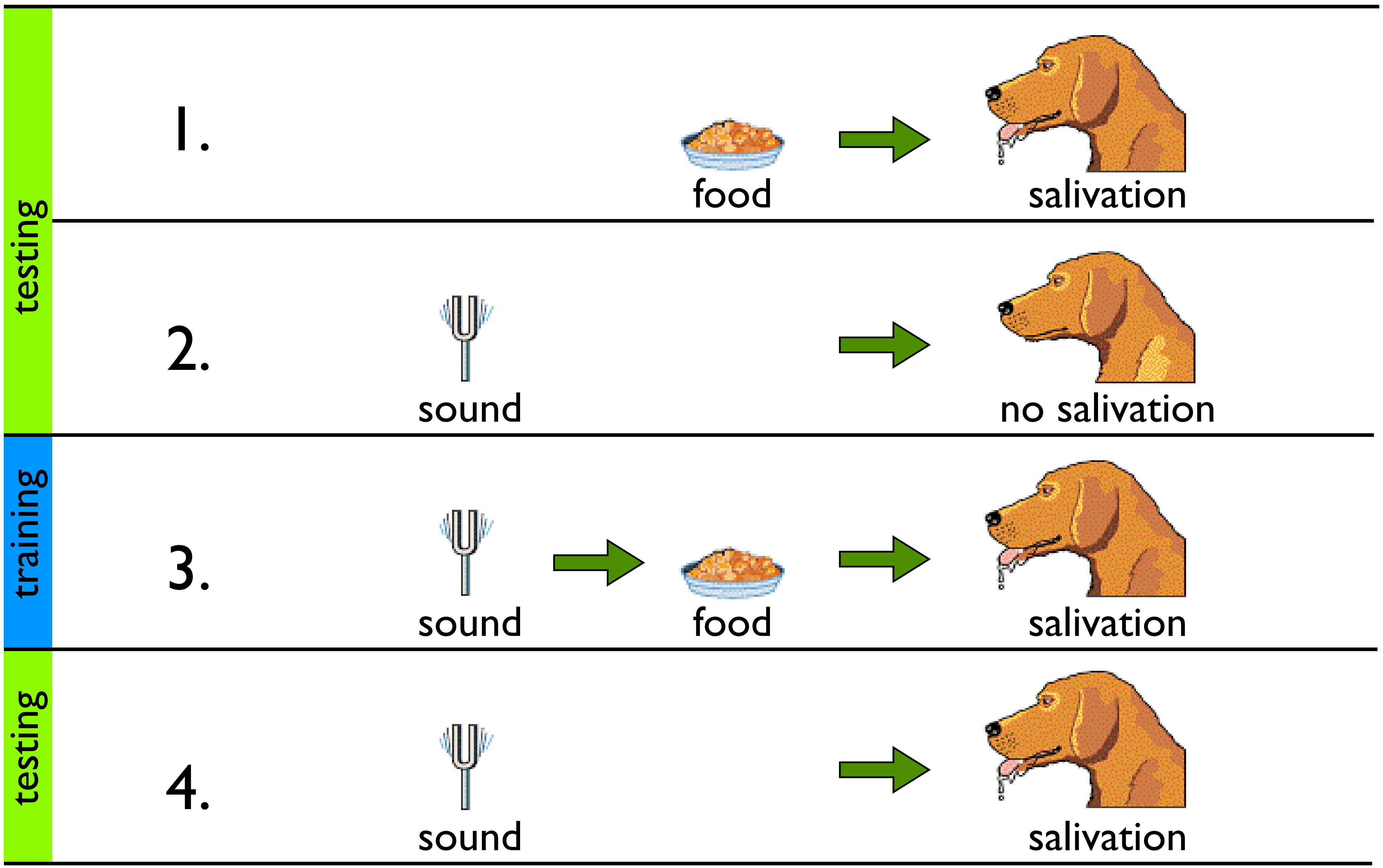
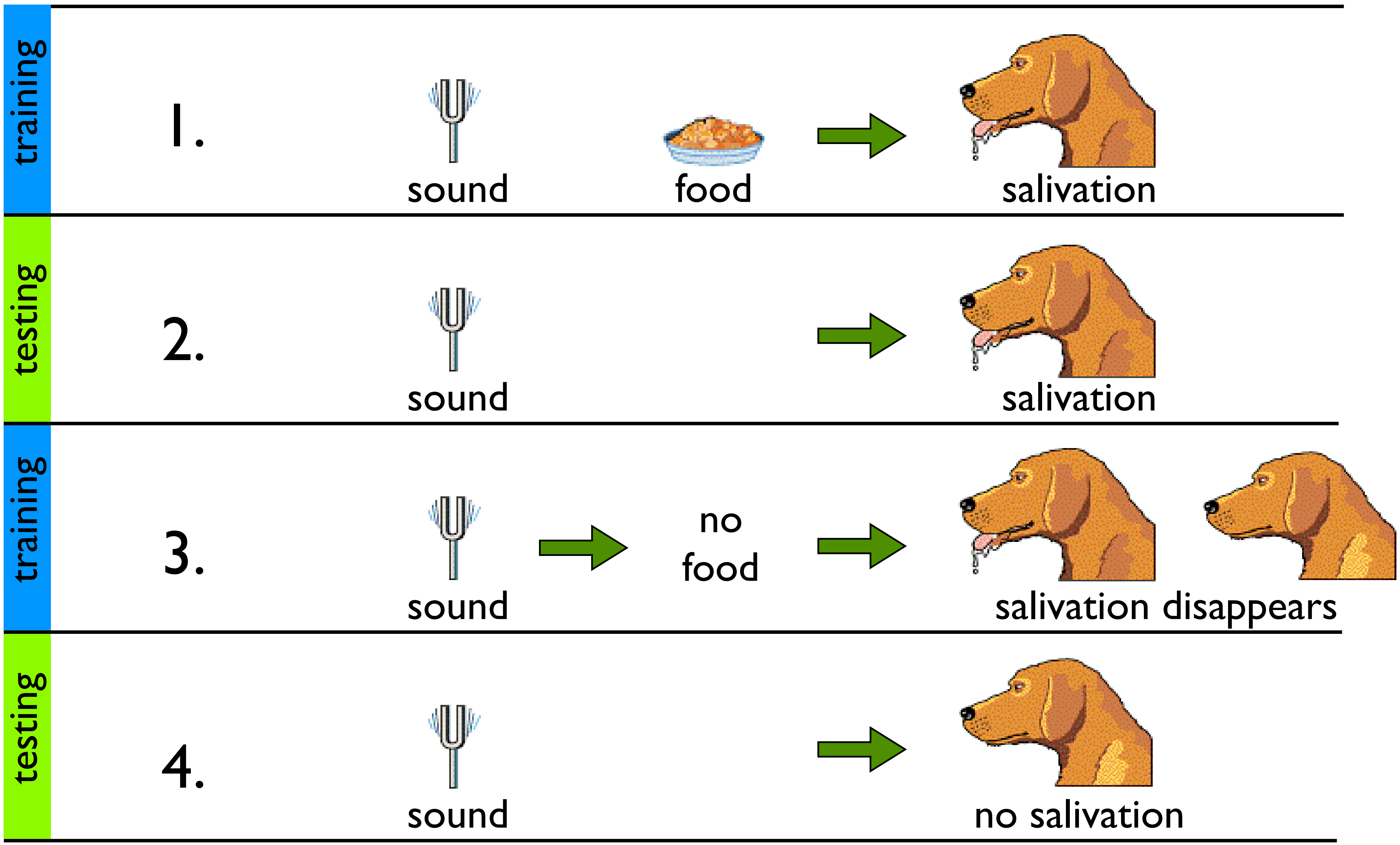
Classical conditioning a la Pavlov



Classical conditioning

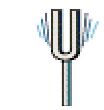


Extinction


training
1.
sound
food
salivation
testing
2.
sound
salivation
training
3.
sound
no
food
salivation disappears
testing
4.
sound
no salivation

What does the dog want?

Assume: The dog wants to be able to predict the reward!

u_i stimulus in trial i: $u_i = 0$ or $u_i = 1$ 

r_i reward in trial i: $r_i = 0$ or $r_i = 1$ 

v_i reward that the dog expects in trial i 

What does the dog learn?

Assume: The dog wants to be able to predict the reward!

u_i stimulus in trial i: $u_i = 0$ or $u_i = 1$ 

r_i reward in trial i: $r_i = 0$ or $r_i = 1$ 

v_i reward that the dog expects in trial i 

Assume: The dog learns to minimize a “loss” function:

$$L = \sum_{i=1}^{N} (r_i - v_i)^2$$

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Assume: dog's model of the world

$$v_i = w u_i$$

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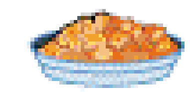
r_i reward in trial i: $r_i = 0$ or $r_i = 1$ 

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parameter that the dog needs to learn from observations

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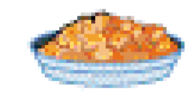
r_i reward in trial i: $r_i = 0$ or $r_i = 1$ 

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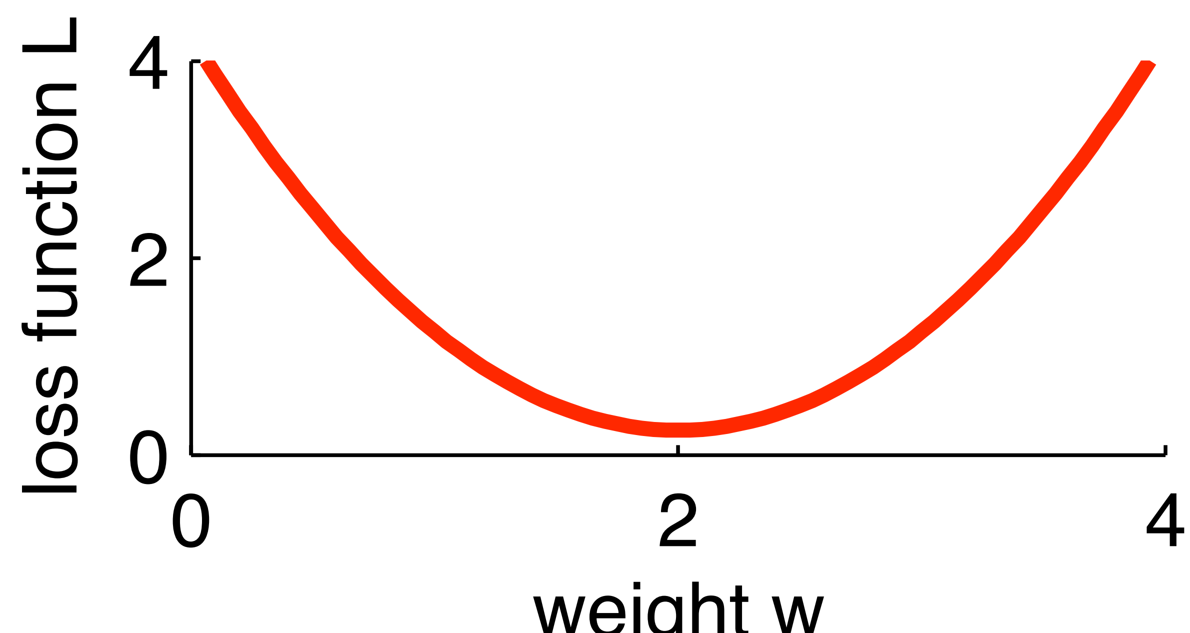
$$v_i = w u_i$$

"Loss" in the i-th trial:

$$L_i = (r_i - w u_i)^2$$



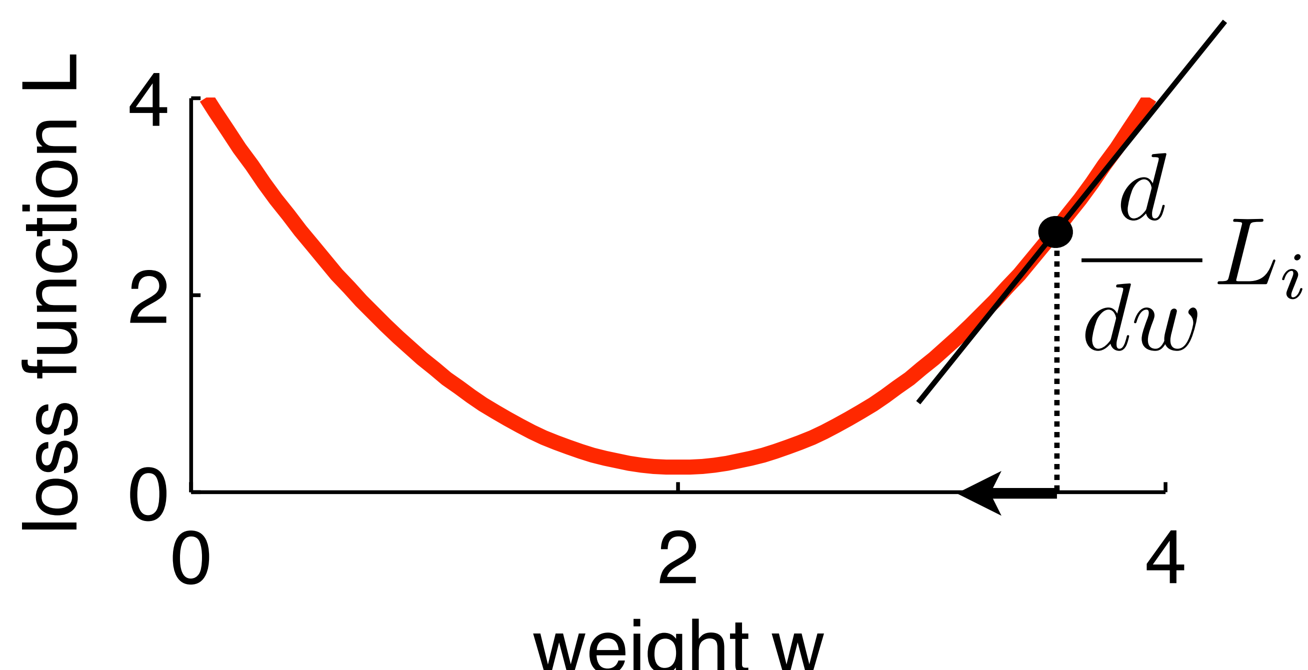
How should the dog adopt its world model?

"Loss" in the i-th trial:

$$L_i = (r_i - wu_i)^2$$

Update parameter w to decrease loss!

$$w \rightarrow w - \epsilon \frac{d}{dw} L_i$$



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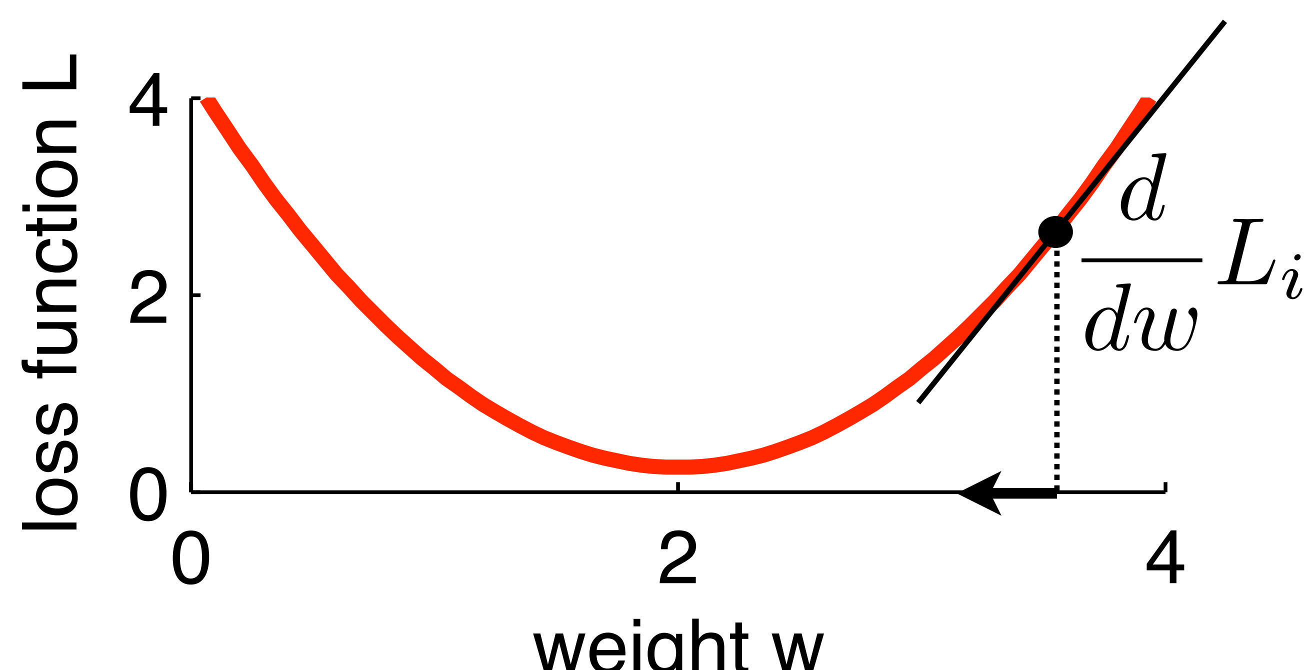

How should the dog adopt its world model?

$$\frac{d}{dw}L_i = \frac{d}{dw}(r_i - wu_i)^2$$

$$= -2u_i(r_i - wu_i)$$

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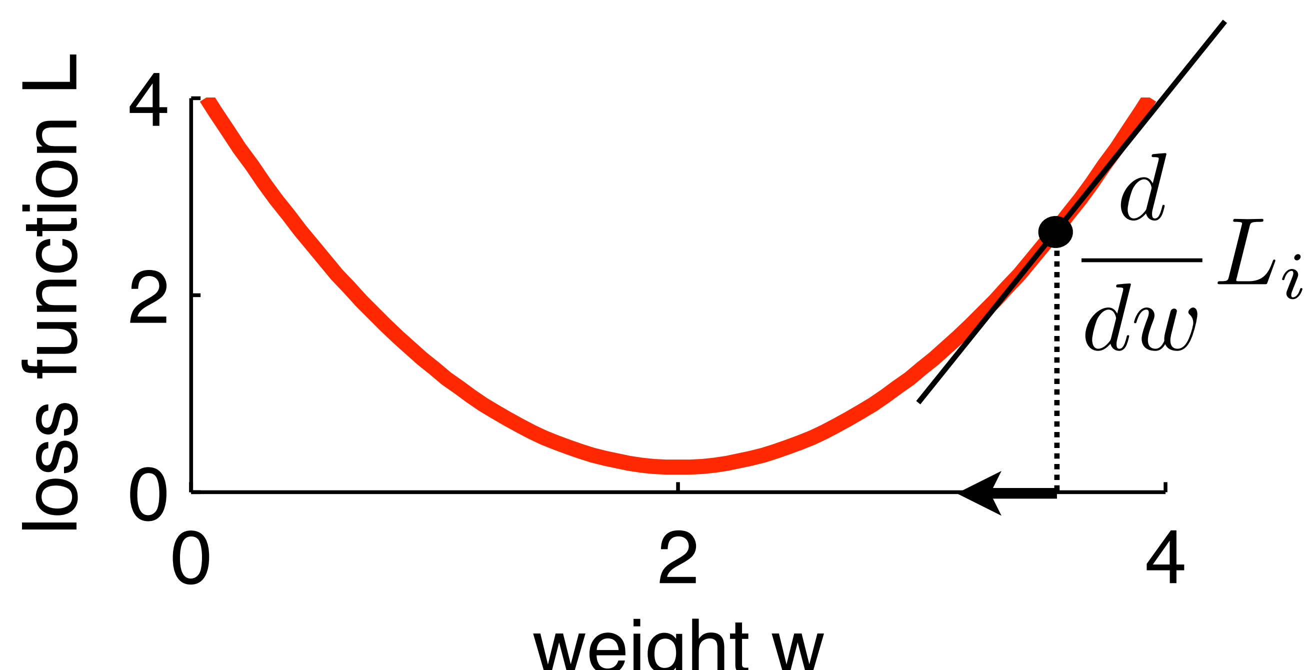


How should the dog adopt its world model?

$$\frac{d}{dw}L_i = \frac{d}{dw}(r_i - wu_i)^2$$

$$= -2u_i(r_i - wu_i)$$

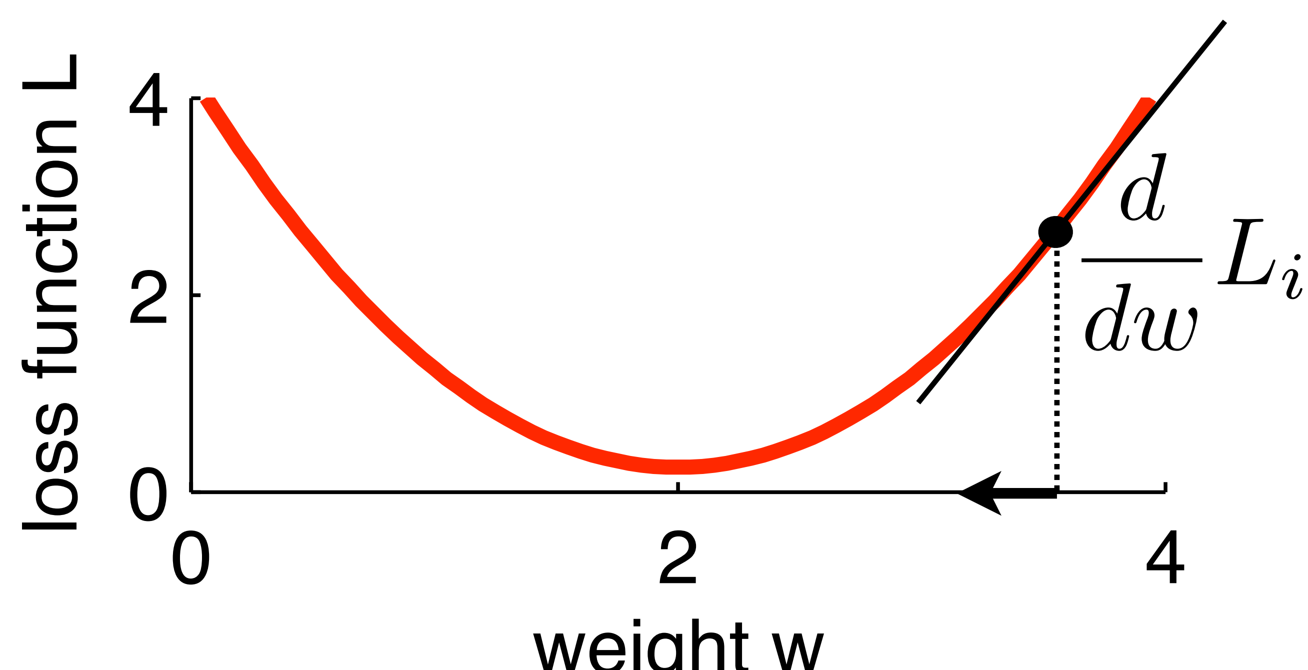
$$= -2u_i\delta_i$$

$$\delta_i = r_i - uw_i = r_i - v_i$$

"prediction error"

Update parameter w to decrease loss!

$$w \rightarrow w - \epsilon\frac{d}{dw}L_i$$



Minimizing the loss function

Minimize loss function (= maximize ability to predict reward)

$$\frac{d}{dw}L_i = \frac{d}{dw}(r_i - wu_i)^2$$

$$= -2u_i(r_i - wu_i)$$

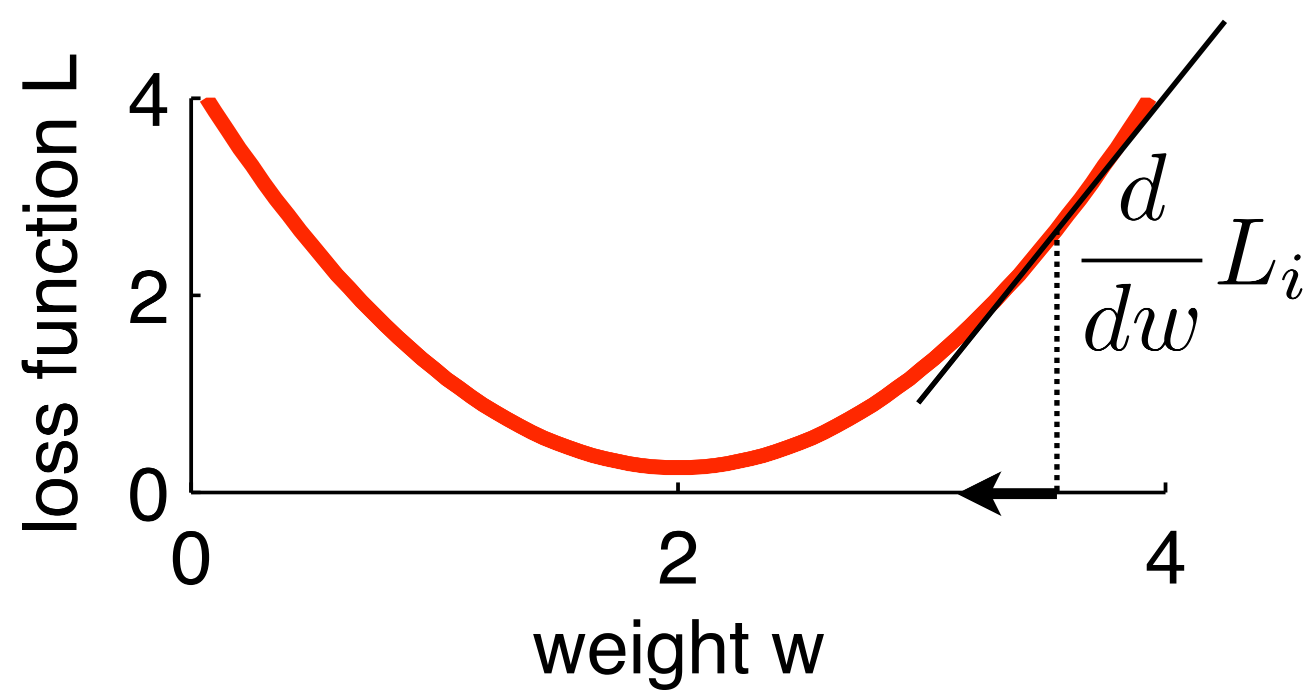
$$= -2u_i\delta_i$$

$$\delta_i = r_i - uw_i = r_i - v_i$$

"prediction error"

"Rescorla-Wagner"-rule

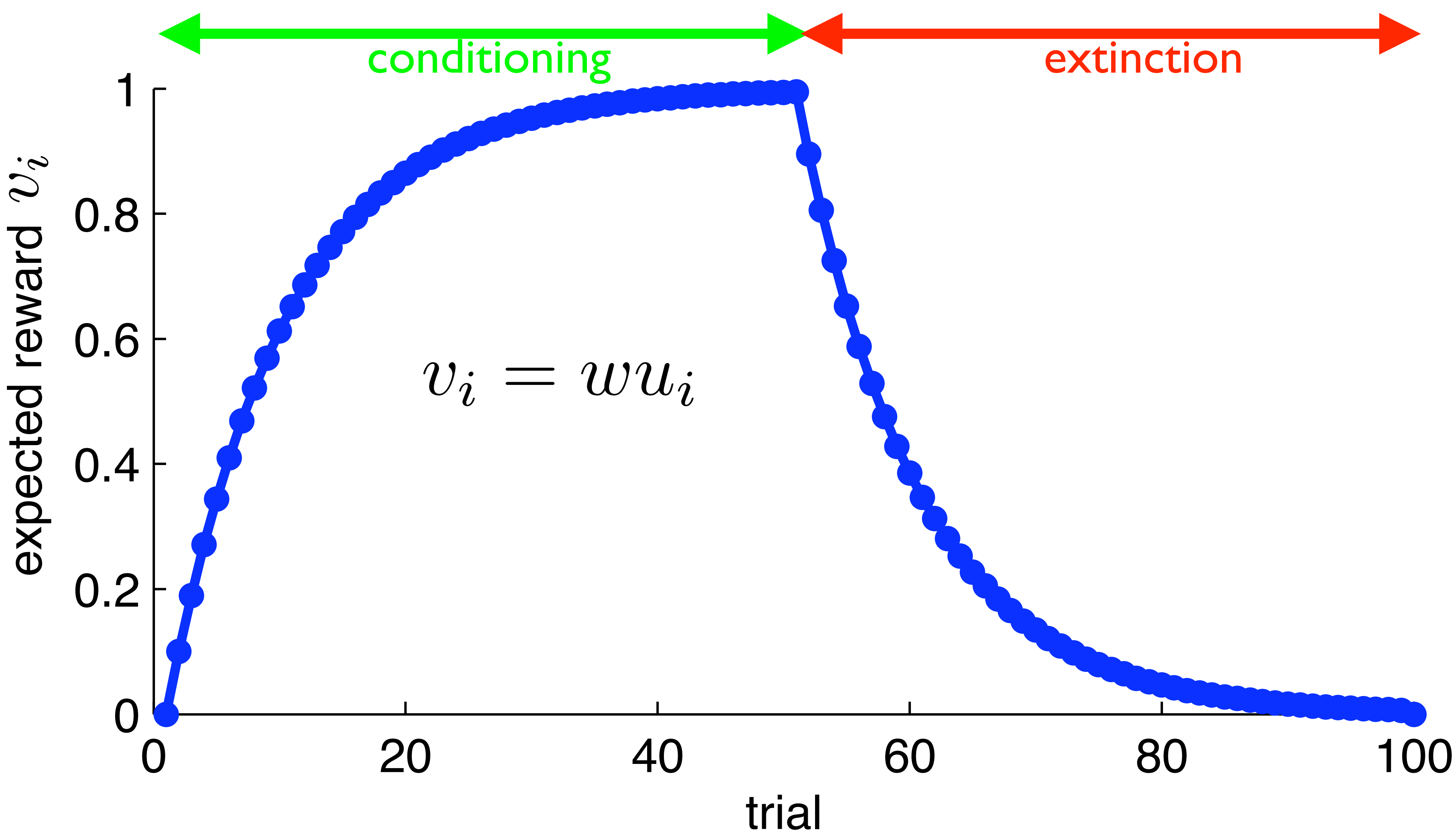
$$w \rightarrow w + \epsilon\delta_i u_i$$



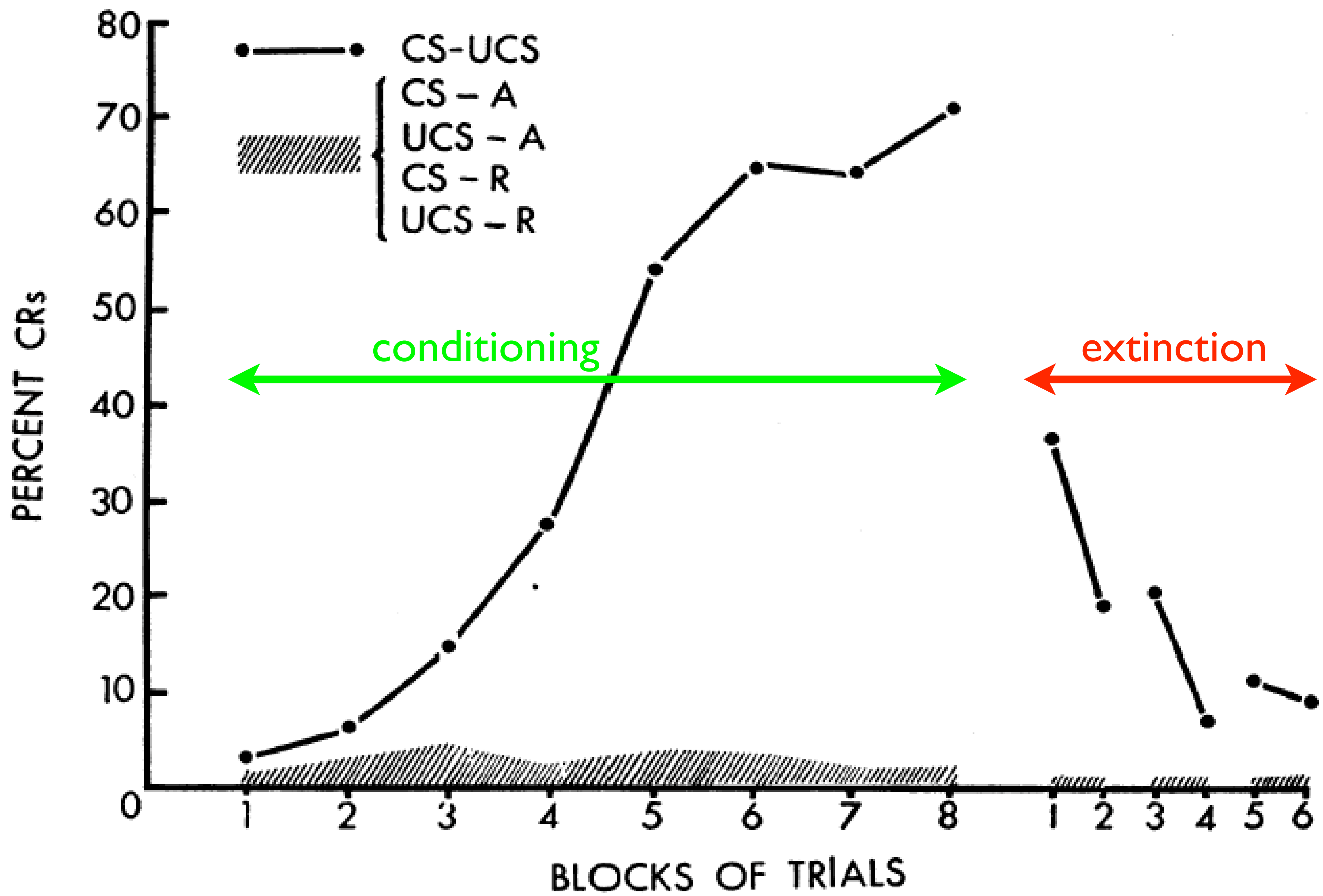
Rescorla-Wagner rule: Conditioning and extinction

stimulus + reward
(CS + US)
$u_i = 1 \quad r_i = 1$

stimulus
(CS)
$u_i = 1 \quad r_i = 0$

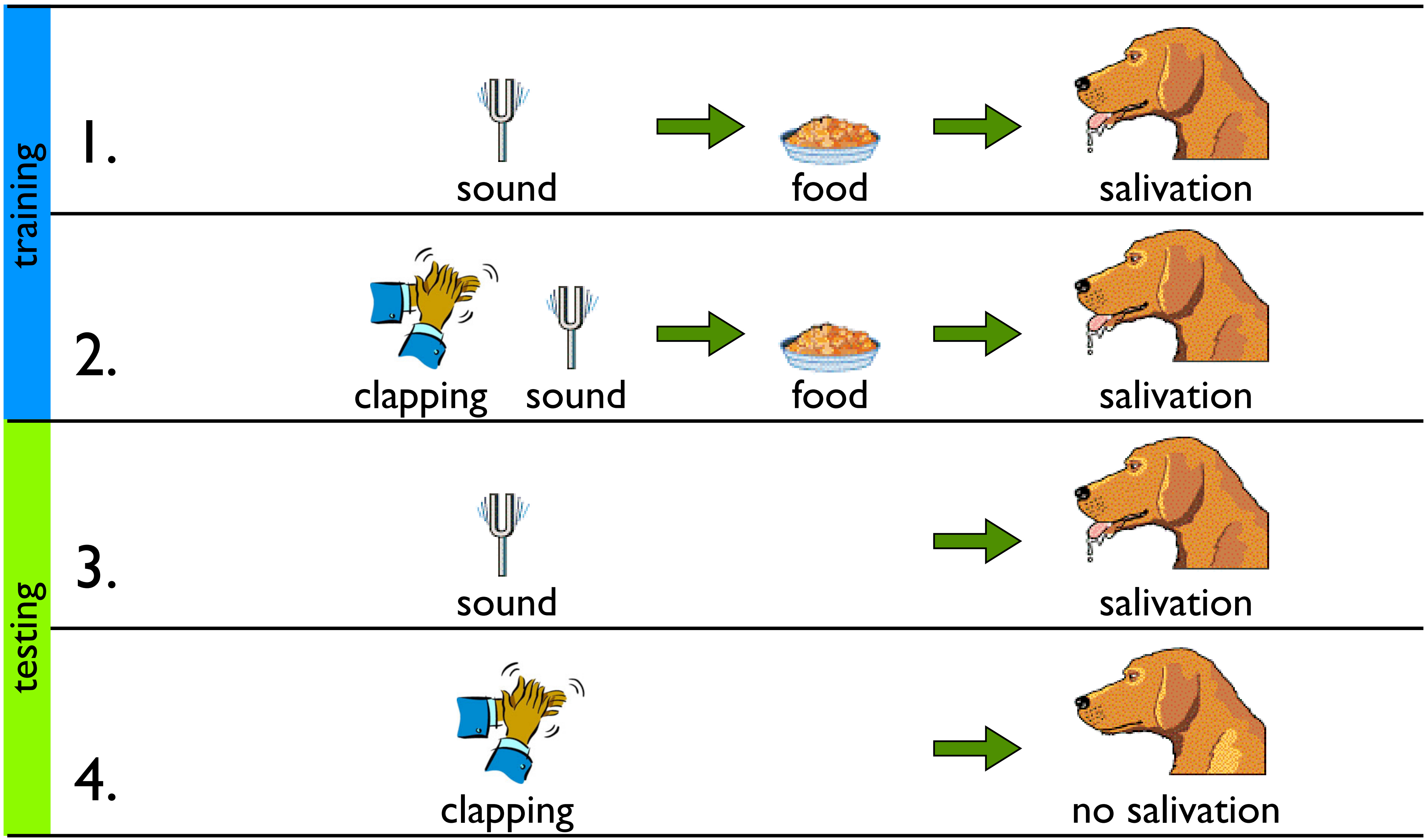


Rabbit eye blinking: Conditioning and extinction



Schneiderman et al, Science, 1962

Classical conditioning: blocking



Reward prediction with multiple stimuli: vectorized Rescorla-Wagner rule

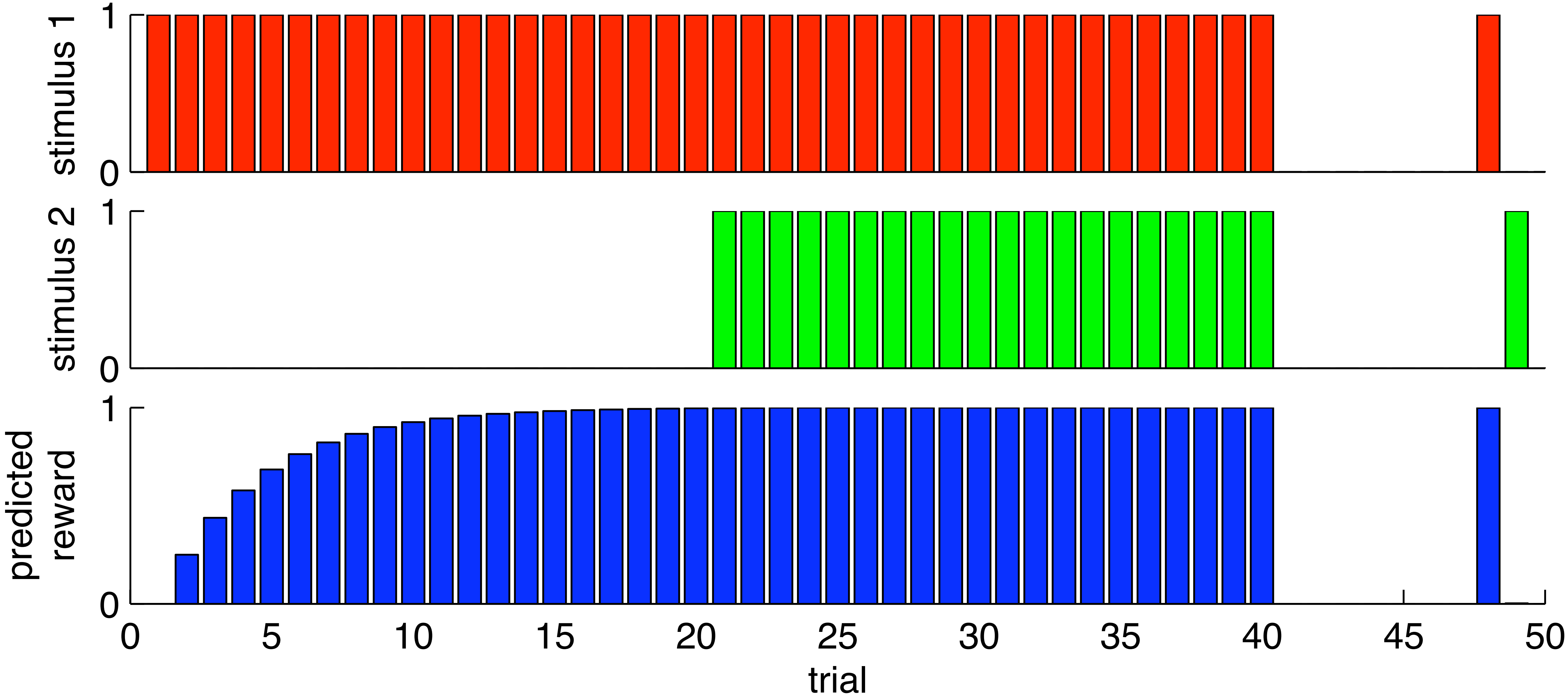
Simple linear model (i-th trial)

$$\underbrace{v_i}_{\text{expected reward}} = \underbrace{\mathbf{w}}_{\text{weight vector}} \cdot \underbrace{\mathbf{u}_i}_{\text{stimulus vector}}$$

"Rescorla-Wagner"-rule

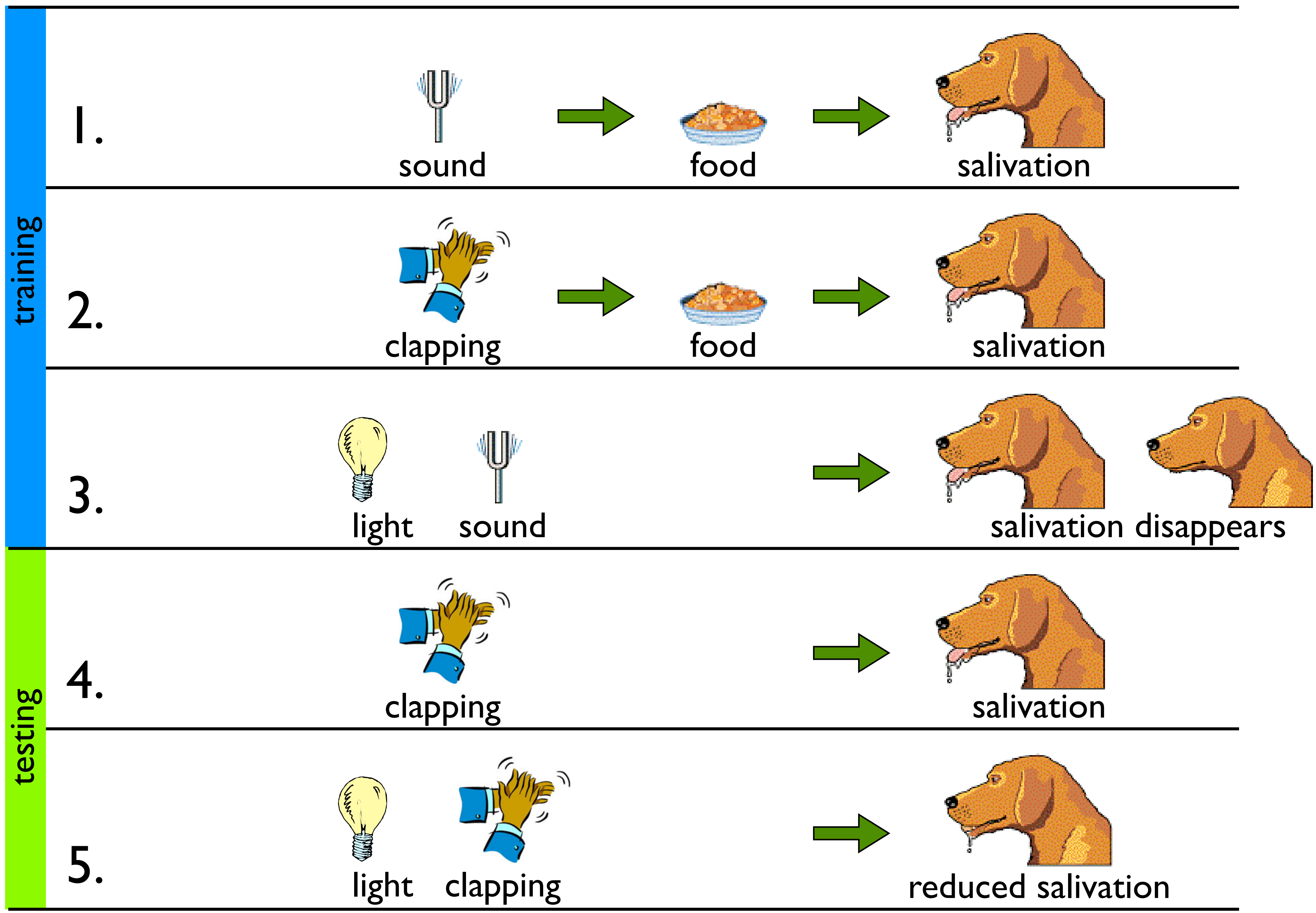
$$\mathbf{w} \rightarrow \mathbf{w} + \epsilon \delta \mathbf{u}_i \qquad \underbrace{\delta = r_i - v_i}_{\text{"prediction error"}}$$

Classical conditioning: blocking

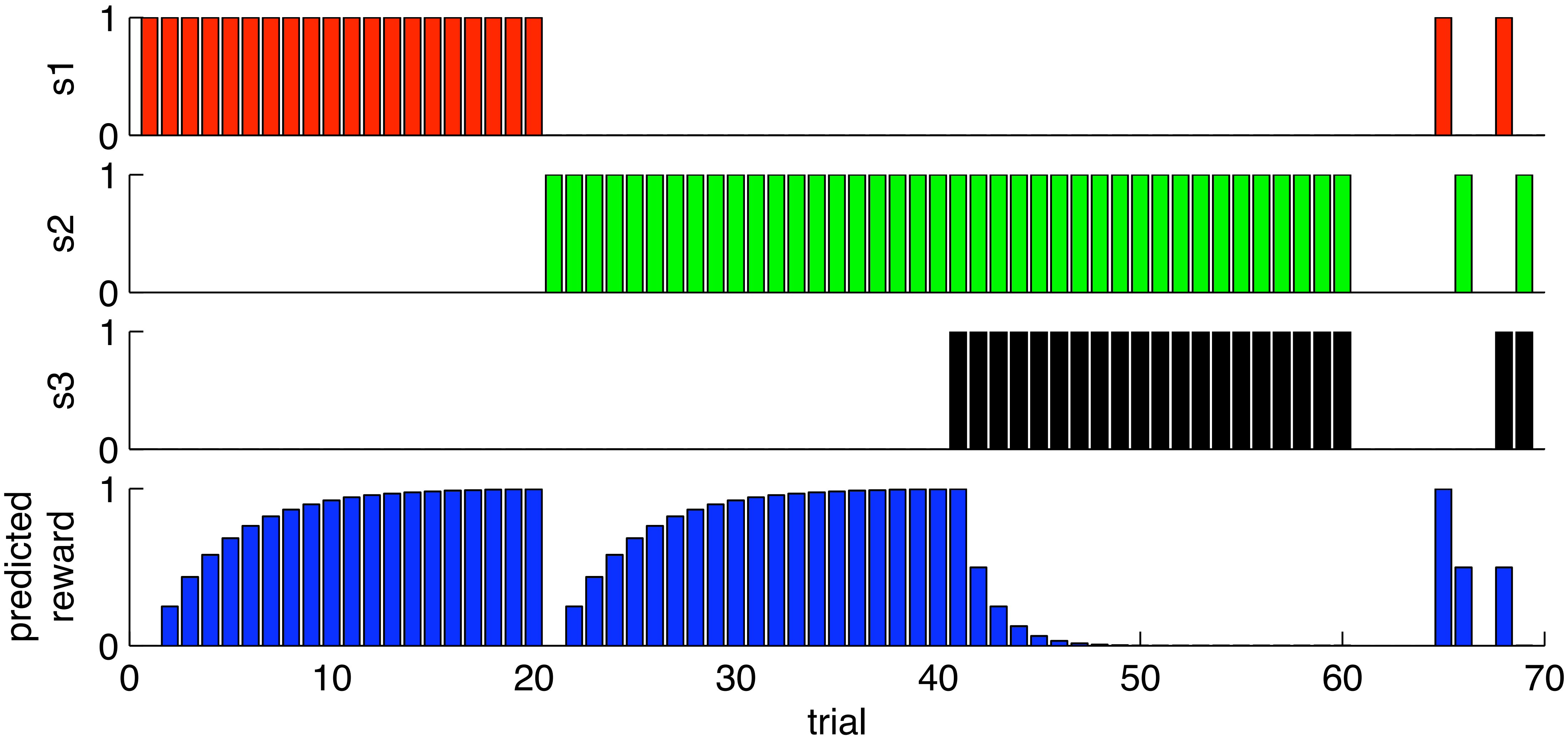


$$v_i = \mathbf{w} \cdot \mathbf{u}_i \qquad \mathbf{w} \rightarrow \mathbf{w} + \epsilon \delta \mathbf{u}_i$$

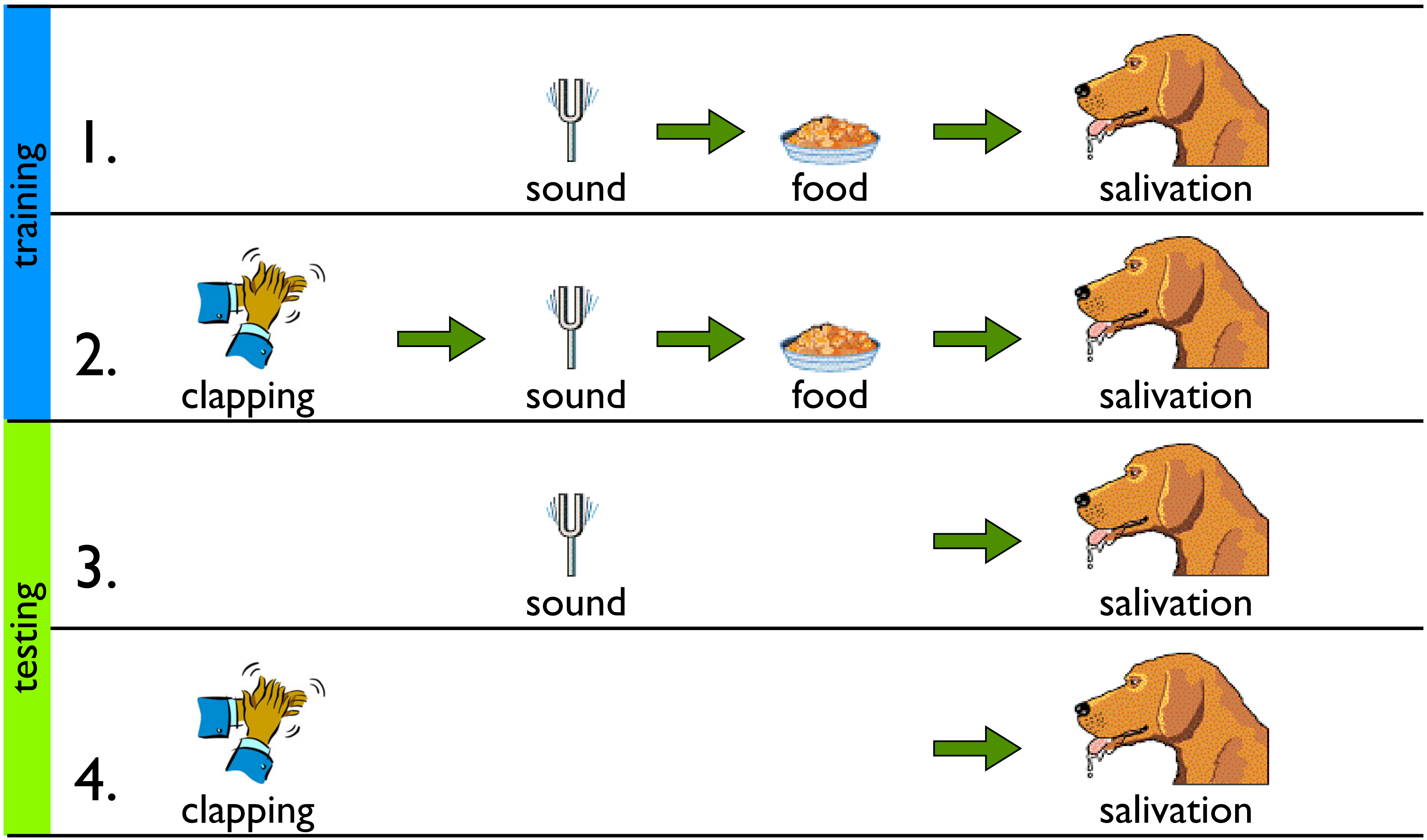
Inhibitory conditioning



Classical conditioning: inhibitory conditioning



Secondary conditioning



→ cannot be explained by Rescorla-Wagner ...