Subjective probabilities

Before we start, a (skinny) primer on probability theory:

- $p(H|D)$ is the conditional probability that H *given that* D.
- The following principles are valid rules for probabilities:

  Quotient Rule $p(H|D) = \frac{p(H \wedge D)}{p(D)}$

  Rule of total probability $p(H) = p(H|D) \times p(D) + p(H|\neg D) \times p(\neg D)$

  Bayes theorem $p(H|D) = \frac{p(D|H) \times p(H)}{p(D)}$

where $\neg$ is a symbol for negation, $\wedge$ a symbol for conjunction.

EXERCISE 1

A ball is drawn blindly from two urns. Urn 1 has one white ball and three black balls. Urn 2 has two white balls and two black balls. It is twice as likely that it is drawn from Urn 2.
A. Is black or white the more probable outcome?
B. A black ball is drawn, what is the probability that it was drawn from Urn 1?

EXERCISE 2

A cab was involved in a hit-and-run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:

- 85% of the cabs in the city are Green, 15% are Blue.
- A witness identified the cab as Blue. The court tested the reliability of the witness under the same circumstances that existed on the night of the accident and concluded that the witness correctly identified each one of the two colors 80% of the time and failed 20% of the time.

What is the probability that the cab involved in the accident was Blue rather than Green (i.e., conditionally on the witness's identification)?

EXERCISE 3 (MONTY HALL)

As a contestant on a TV game show, you are invited to choose any one of three doors and receive as a prize whatever lies behind it - i.e., in one case, a car, or, in the other two, a goat. When you have chosen, the host opens one of the other two doors, behind which he knows there is a goat, and offers to let you switch your choice to the third door. Would that be wise?

EXERCISE 4

In the fall of 1973, when 8442 men and 4321 women applied to graduate departments at U. C. Berkeley, about 44% of the men were admitted, but only about 35% of the women. It looked like sex bias against women. But when admissions were tabulated for the separate departments - as below, for the six most popular departments, which together accounted for over a third of all the applicants - there seemed to be no such bias on a department-by-department basis.

Department A admitted 62% of 825 male applicants, 82% of 108 females.
Department B admitted 63% of 560 male applicants, 68% of 25 females.
Department C admitted 37% of 325 male applicants, 34% of 593 females.
Department D admitted 33% of 417 male applicants, 35% of 375 females.
Department E admitted 28% of 191 male applicants, 24% of 393 females.
Department F admitted 6% of 373 male applicants, 7% of 341 females.

Actually the tabulation suggested an innocent one-sentence explanation of the overall statistics. *What was it?* (Hint: What do the statistics indicate about how hard the different departments were to get into?)