Thinking about Truth & Befief

Exercise I : truth conditions

Consider the following sentences:

(1) John is happy and Marry did not come
(2) It is not the case that John is tall and Bob is tall
(3) If Mary comes, then John is not happy
(4) John is tall and John is not tall
(5) It is not the case that John is tall and John is not tall

A. For each of them, describe the circumstances under which they will come out true, and the circumstances under which they will come out false, that is describe what their truth-conditions are.

Example:
Sentence (1) can be analyzed as “p or not q”, that is “p $\wedge$ ¬q”, with p = John is happy and q = Marry came. Using the truth-tables, we get:

p	q	¬q	p $\wedge$ ¬q
V	V	F	F
V	F	V	V
F	V	F	F
F	F	V	F

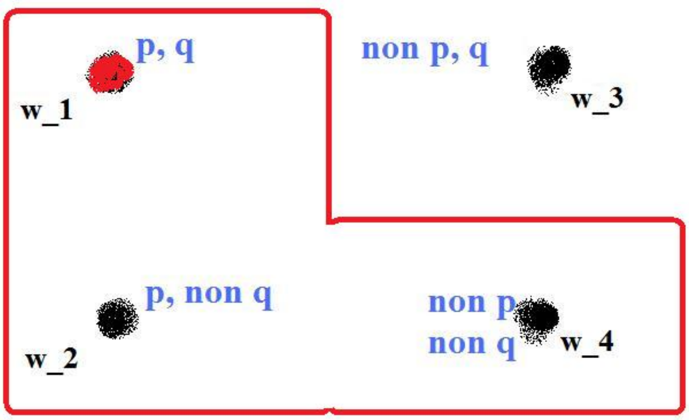
B. On the basis of your answer to question A, suggest a threefold partitions of sentences according to their truth-conditions.

Exercise II : beliefs and iterated beliefs

We have provided a truth clause for the belief operator *B* based on a function *f*(*w*) which associates to every possible world *w* the set of worlds *f*(*w*) which are considered as possible from the perspective of *w*:

w |= Bφ iff for all w’ in *f*(*w*), w’ |= φ

A. Let us consider the following space of worlds as before:



(the red line is meant to indicate that *f*(w_1)={w_1,w_2,w_4}).

D. Bonnay

Say whether the following claims are true:

w_1 |= Bp
w_1 |= Bq
w_1 |= B(p --> q)
w_1 |= B(q --> p)

B. Give a model and a world in that model such that, at that world, an agent believes p but does not believe that he believes p. [giving a model consists in giving a set of worlds, and for each world *w* in that set, saying whether p is true at *w* and what is *f*(*w*)]

C. An agent's beliefs are said to satisfy positive introspective iff whenever the agent believes something, he also believes that he believes it. This property of belief gets reflected in our models by a corresponding property of the function *f*. Find necessary and sufficient conditions on *f* to ensure that Bp --> BBp. [Question C is more difficult]

Exercise III : the preface paradox (adapted from Foley)

You write book, say, a cognitive science book. In this book, you make many assertions, each of which you can adequately defend. In particular, suppose that it is rational for you to have a degree of confidence *x* or greater in each of these propositions, where *x* is sufficient for belief but less than 1. Nonetheless, you admit in the preface that you are not so naïve as to think that your book contains no mistakes. You understand that any book as ambitious as yours is likely to contain at least a few errors.

A. Show intuitively (without providing an explicit probabilistic model) that if beliefs can be aggregated, you are bound to entertain contradictory beliefs.

B. Give a probabilistic model of the preface scenario. Let p_i be the i^{th} sentence in the book and ψ be $\neg (p_1 \wedge p_2 \wedge \ldots \wedge p_n)$ where n is the number of sentences in the book. Compute the values of $Pr(p_i)$ and $Pr(\psi)$. Show how this matches intuitions discussed as an answer to A.

C. On the basis of the model you gave as an answer to B, say what happens to $Pr(p_i \mid \chi)$ and $Pr(\psi \mid \chi)$ when χ varies.

D. Using your answer to question C., suggest an alternative to the Lockean thesis which is able to escape the preface paradox. Hint: give a characterization of belief in terms of subjective probability *which captures the idea that beliefs should be stable enough under conditionalizing*.